

# Optimizing the Updating Stopping Condition for a Predictive Maintenance Policy

Ming-Yi You \*

No. 36 Research Institute of CETC, No. 387 Hong Xing Rd., Jia Xing, Zhe Jiang Province, 314033, China

**Abstract:** The updating stopping condition (USC) has great impact on the effectiveness of a predictive maintenance (PdM) policy, but did not receive enough attention. This paper reviews the common USCs, proposes a residual life based USC, and evaluates the influence of the USCs on the effectiveness of a PdM policy. The commonly used USCs are concretely defined in a PdM policy based on the stochastic linear degradation model. An extensive numerical investigation compares the performances of the PdM policy using different USCs. The investigation results verify the importance of optimizing the USC for a PdM policy.

**Keywords:** Updating stopping condition, predictive maintenance, maintenance optimization, reliability.

## 1. INTRODUCTION

The economic relevance of maintenance in all sectors of industrial sectors is continuously promoting the advance of maintenance technology and management [1]. Nowadays, maintenance policies such as condition-based maintenance (CBM) and predictive maintenance (PdM) are important research subjects in reliability engineering. Due to the systematic collection of system condition information, reasonable prediction of future development of system condition, and a wise balance between maintenance cost and risk of system failure, PdM can effectively improve asset reliability, reduce maintenance cost, extend useful lifetimes and reduce safety risks [2-4]. Different from the population-oriented preventive maintenance (PM) policies and CBM policies, a PdM policy optimizes the maintenance schedule based on the system's condition information monitored in real time. Consequently, the optimized maintenance schedule in a PdM policy is only applicable to the specific system being monitored. In addition, the PdM policy updates the maintenance schedule with the arrival of the system's most recent condition monitoring information. Consequently, a PdM policy needs an updating stopping condition (USC) to stop the update of the maintenance schedule and decides the final maintenance time.

An updating step length based USC is proposed for a component-level sequential PdM policy in [5]. Based on the proposed USC, the update of the maintenance schedule is stopped once the distance between the maintenance schedule and the current moment is less than the updating step length. Kaiser and Cebrael

used a reliability based USC in a component-level PdM policy based on an updating system degradation model [6]. Curcuro, Galante and Lombardo decided the updating stopping moment through comparing the expected maintenance cost in the considered time interval with and without a PM action in each decision moment [7]. Zio and Compare proposed to perform a PM act when the current moment is the optimized maintenance schedule or the system's condition reaches the safety threshold with a given probability [1].

Although many researchers have used the USCs in different PdM policies, most of the studies focus on the development of PdM models and the optimization of maintenance schedule. Actually, a USC is a key factor which decides the final maintenance time and essentially influences the effectiveness of a PdM policy. Stopping the update of maintenance schedule when the system is still normally operating is ineffective, as the condition monitoring information closer to system failure is simply ignored when deciding the maintenance schedule. On the other hand, an overaggressive USC could lead to system failure before maintenance acts due to the rapid change of system condition. An ideal USC should make a wise balance between scheduling maintenance acts using more recent system conditions and the risk of unexpected system breakdowns. By the above motivations, this paper focuses on the effect of USCs on the effectiveness of PdM policies and their optimization. In addition, an extended updating step length based USC and a novel residual life based USC is proposed to be valuable alternates in the USC library.

The rest of the paper is organized as follows: Section II reviews the commonly used USCs, extends the updating step length based USC and proposes a

\*Address correspondence to this author at the No. 36 Research institute of CETC, No.387 Hong Xing Rd., Jia Xing, Zhe Jiang Province, 314033, China; Tel: 86-0573-83678032; Fax: 86-0573-83683600; E-mail: youmingyi@126.com

residual life based USC. Section III gives an example PdM policy, concretizes the USCs in the given PdM policy. Section IV evaluates the USCs based on an extensive simulation study. Finally, section V concludes the paper.

## 2. USCS FOR PDM POLICIES

This section reviews the commonly used USCs in PdM policies with some extensions. It is noted that the mentioned USCs are primarily proposed for single-unit systems, the relationship between units in multi-unit systems are not strictly considered, which may suggest group maintenance.

The 1<sup>st</sup> commonly used USC is the updating step length based USC [5], and can be given as:

$$T^*(t) - t \leq \Delta t \quad (1)$$

where  $t$  is the current moment,  $T^*(t)$  is the PM schedule suggested by the PdM model closest to the current moment,  $\Delta t$  is the updating step length or the sampling interval of condition monitoring. The USC in Eq. (1) suggests updating stopping and PM acts at time  $t$  when the distance between the current moment and  $T^*(t)$  is smaller than the updating step length. When the degradation process becomes noisy and the predictions of the system's future health become very uncertain, the USC in Eq. (1) may easily lead to abrupt system breakdown. To avoid the unexpected failure, the USC in Eq.(1) can be extended as

$$T^*(t) - t \leq n \cdot \Delta t \quad (2)$$

where  $n$  is a positive integer.

The 2<sup>nd</sup> commonly used USC is the reliability based USC. Kaiser and Gebraeel proposed a reliability based USC as follows [6]:

$$\min_{0 < t_k < \infty} \left\{ 1 - \left[ F_T(t_{k+\delta}) - F_T(t_k) \right] \right\} \geq R \quad (3)$$

where  $t_k$  and  $t_{k+\delta}$  are time variable,  $\delta$  is a small time interval, and  $t_{k+\delta} > t_k$ ,  $F_T(t_k)$  is the cumulative distribution function of system failure at time  $T$  predicted at time  $t_k$ ,  $R$  is the ideal probability threshold. The interpretation for the USC in Eq. (3) is to stop updating of maintenance schedule when the failure probability in a time interval  $(t_k, t_{k+\delta})$  exceed a certain value. In practice, people may be more concerned with the reliability level of the system, and the reliability based USC can be in the form as follows:

$$1 - F_T(T^*(t)) \leq R' \quad (4)$$

where  $R'$  is the reliability threshold. The interpretation for the USC in Eq. (2) is to stop updating of maintenance schedule when the expected system reliability at time  $T^*(t)$  is smaller than the ideal reliability level  $R'$ .

The 3<sup>rd</sup> commonly used USC is the condition based USC. Zio and Compare suggested performing the PM acts when the probability of system condition to be worse than the safety level is larger than a certain threshold [1]. They estimated the system condition using particle filtering. In a PdM policy, the condition based USC can be

$$p(s(t) \leq s_r) \geq r_s \quad (5)$$

where  $s(t)$  the system condition at current time  $t$ ,  $s_r$  is the condition level,  $r_s$  is the probability threshold for the system's condition. The USC in Eq. (5) means to stop updating and perform a PM act at time  $T^*(t)$  when the probability of the system's condition to be worse than  $s_r$  is larger than  $r_s$ . To use the USC in Eq. (5), the mapping function between the system's monitored performance variables and its health state should be defined. If the system's performance variables can directly reflect its health state, then the USC in Eq. (5) can be reduced as

$$v_i(t) \leq s_{ir} \quad (6)$$

where  $v_i(t)$  is the system's  $i^{\text{th}}$  performance variable, and  $s_{ir}$  is the corresponding condition level. It is noted that Eq. (6) is applicable to the case of the greater  $v_i(t)$  the better. In the case of the smaller  $v_i(t)$  the better, Eq. (6) should be revised as

$$v_i(t) \geq s_{ir} \quad (7)$$

In some cases, the residual lifetime of a system may be a more understandable variable to maintenance engineers or manufacturing managers, as it directly tells the time interval the system can continue to survive. In such cases, a residual life based USC is proposed as

$$RL(t) \leq r_l \quad (8)$$

where  $RL(t)$  is the residual life of the system predicted at time  $t$ ,  $r_l$  is the lifetime threshold.

### 3. PDM POLICY

This section gives a PdM policy widely used in manufacturing/production industry and concretizes the USCs. The established PdM policy constitutes the base for further numerical analysis.

This paper establishes a component-level PdM policy based on a stochastic linear degradation model. The key factors for the PdM policies are: 1) Objectives: minimizing the maintenance cost rate within a replacement cycle; 2) Maintenance policy: sequential maintenance; 3) maintenance effect: perfect PM and corrective maintenance (CM); 4) Degradation characteristics: stochastic liner degradation model; 5) maintenance constraints: none. The timing of the PdM policy is illustrated in Figure 1.

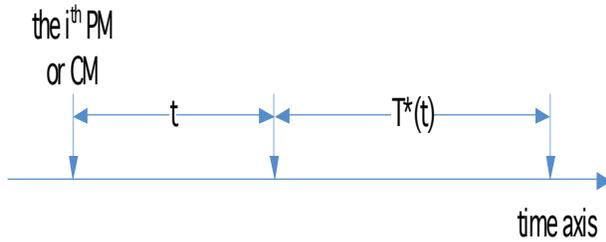


Figure 1: Timing of the PdM policy.

Based on the above factors, the maintenance cost rate within a replacement cycle estimated at time  $t$  in the PdM policy is

$$r(t) = \frac{C_{CR}(t) + C_{PR}(t)}{T_o(t)} \quad (9)$$

where  $r(t)$  is the maintenance cost rate within the replacement cycle estimated at time  $t$ ,  $C_{CR}(t)$  is the CM cost within the replacement cycle estimated at time  $t$ ,  $C_{PR}(t)$  is the PM cost within the replacement cycle estimated at time  $t$ ,  $T_o(t)$  is the operation time within the replacement cycle estimated at time  $t$ .

In Eq. (9),  $C_{CR}(t)$  could be estimated as

$$C_{CR}(t) = CCR \cdot [1 - R(T(t)|t)] \quad (10)$$

where CCR is the maintenance cost per CM,  $T(t)$  is the PM schedule estimated at time  $t$ ,  $R(T(t)|t)$  is the system reliability at time  $T(t)$  predicted based on its condition monitoring information up to time  $t$ . On the other hand,  $C_{PR}(t)$  in Eq.(9) can be concretized as

$$C_{PR}(t) = CPR \cdot R(T(t)|t) \quad (11)$$

where CPR is the maintenance cost per PM. In Eq.(9),  $T_o(t)$  could be estimated as

$$T_o(t) = t + \int_t^{T(t)} R(v|t) dv \quad (12)$$

where  $R(v|t)$  is the system reliability at time  $v$  predicted based on its condition monitoring information up to time  $t$  and  $v \geq t$ . At time  $t$ , to find the optimal  $T(t)$  (denoted as  $T^*(t)$ ) that minimizes  $r(t)$ , the estimation method for  $R(v|t)$  must be clearly defined.

This paper considers a single-unit system with one performance variable, characterizes the performance variable using the stochastic linear degradation model, and further calculates  $R(v|t)$ . Examples for the single-unit system with one performance variable modeled by the stochastic linear degradation model include rolling bearings [8], laser systems [9], thrust drillers [10], etc. The stochastic linear degradation model is [11]:

$$L(t) = \theta + \beta t + \varepsilon(t) \quad (13)$$

where  $L(t)$  is the monitored performance variable at time  $t$ ,  $\theta$  is a constant and  $\theta \sim N(\mu_0, \sigma_0^2)$ , where  $N(\mu_0, \sigma_0^2)$  means normal distribution with mean  $\mu_0$  and variance  $\sigma_0^2$ ,  $\beta$  reflects the changing rate of the system condition and  $\beta \sim N(\mu_1, \sigma_1^2)$ ,  $\varepsilon(t)$  is an error term which follows centralized Brownian motion and  $\varepsilon(t) \sim N(0, \sigma^2 t)$ . In the operation process of the system, the most recent condition monitoring information arrives at each condition sampling point. Gebraeel etc. proposed a Bayes based method for updating the model parameters in Eq. (13) [11]. Based on the updated model parameters,  $R(v|t)$  can be predicted as

$$R(v|t) = P(L(v|t) \leq CL) = \Phi\left(\frac{CL - \tilde{\mu}_1(v)}{\tilde{\sigma}_1(v)}\right) \quad (14)$$

where  $P(A)$  means the probability of event A,  $L(v|t)$  is the system's performance variable at time  $v$  predicted based on its condition monitoring information up to time  $t$ , CL is the critical level for the performance variable and the system is considered failed once its performance variable exceeds CL.  $\Phi(\cdot)$  is the cumulative distribution function of standard normal

distribution.  $\tilde{\mu}_1(v)$ ,  $\tilde{\sigma}_1(v)$  is the predicted values of  $\mu_1$  and  $\sigma_1$  and are given as:

$$\begin{aligned} \tilde{\mu}(v) &= L(t) + \mu_\beta(v-t) \\ \tilde{\sigma}(v) &= \sqrt{\sigma^2(v-t) + \sigma_\beta^2(v-t)^2} \end{aligned} \quad (15)$$

where  $\mu_\beta$  and  $\sigma_\beta$  are respectively the updated values of  $\mu_1, \sigma_1$  at time t. The details of the updating method of model parameters can be referred to [11]. Substituting Eqs. (10) ~ (12) and Eq. (14) into Eq.(9) gives:

$$r(t) = \frac{CCR \cdot \left[ 1 - \Phi \left( \frac{CL - L(t) - \mu_\beta(T(t) - t)}{\sqrt{\sigma^2(v-t) + \sigma_\beta^2(v-t)^2}} \right) \right] + CPR \cdot \Phi \left( \frac{CL - L(t) - \mu_\beta(T(t) - t)}{\sqrt{\sigma^2(v-t) + \sigma_\beta^2(v-t)^2}} \right)}{t + \int_t^{T(t)} \Phi \left( \frac{CL - L(t) - \mu_\beta(v-t)}{\sqrt{\sigma^2(v-t) + \sigma_\beta^2(v-t)^2}} \right) dv} \quad (16)$$

Based on Eq. (16),  $T^*(t)$  is given as:

$$T^*(t) = \arg \min_{T(t)} [r(t)] \quad (17)$$

By Eq. (17), there is a temporary  $T^*(t)$  at each condition monitoring moment, and consequently a USC is needed to stop updating and determine the final PM time. The updating step length based USC is clearly defined in Eq. (2). For reliability based USC defined in Eq. (4), the USC for the PdM policy can be more concretely written as:

$$\Phi \left( \frac{CL - L(t) - \mu_\beta(T^*(t) - t)}{\sqrt{\sigma^2(T^*(t) - t) + \sigma_\beta^2(T^*(t) - t)^2}} \right) \leq R' \quad (18)$$

The PdM policy is established based on the fact that the performance variable can directly reflect the system's health state, and hence the condition based USC is

$$L(t) \geq s_r \quad (19)$$

In addition, the residual life based USC can be concretized as

$$\int_t^\infty \Phi \left( \frac{CL - L(t) - \mu_\beta(v-t)}{\sqrt{\sigma^2(v-t) + \sigma_\beta^2(v-t)^2}} \right) dv \leq r_t \quad (20)$$

#### 4. NUMERICAL INVESTIGATION

This section conducts an extensive numerical investigation on the effect of the USCs on the effectiveness of the typical PdM policy given in Section III, and discusses the problem of USC optimization.

Table 1: Model Parameters

parameter	$\mu_0$	$\sigma_0$	$\mu_1$	$\sigma_1$	$\sigma$	CL
value	1	1	4	1	1/2/4	200

The 1<sup>st</sup> step of the numerical investigation is data generation, and failure definition. The stochastic linear

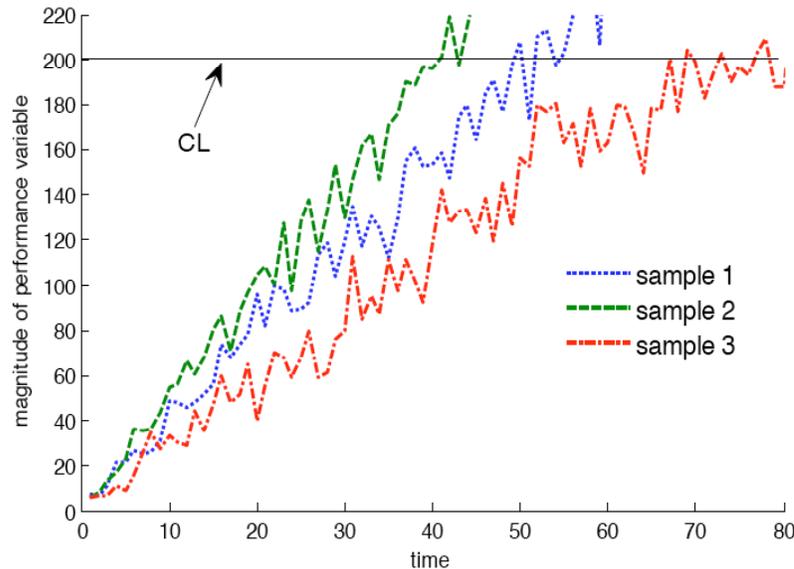


Figure 2: 3 Randomly selected degradation processes with  $\sigma = 2$ .

degradation model is used to simulate the performance variables in the system degradation processes. The model parameters are defined in Table 1 without loss of generality. In addition, the CL is also defined in Table 1. Using the model parameters in Table 1, 100 degradation processes are simulated. Figure 2 illustrates 3 degradation processes randomly selected out of the simulated samples.

The 2<sup>nd</sup> step of numerical investigation is to establish the PdM model in Eq.(17) and calculate  $T^*(t)$  for each simulated sample and at each condition monitoring point. Without loss of generality,  $\Delta t = 1$  is used in this paper. To establish the PdM model, the maintenance cost rate  $CCR/CPR$  needs to be defined at first. Herein, two typical cases for  $CCR/CPR = 2$  and  $CCR/CPR = 4$  are considered.

The 3<sup>rd</sup> step of numerical investigation is to determine the maintenance schedule for each sample based on different USCs and analyze the performance of each USC based on the performance of the PdM using the USC. The performance of the PdM can be defined as

$$r_A = \frac{PPR \cdot n_{PM} + CPR \cdot n_{CM}}{\sum_{i=1}^{100} T_{Oi}} \quad (21)$$

where  $r_A$  is the actual average maintenance cost rate for the simulated samples,  $n_{PM}$  is the actual number of samples receiving PM acts before failure, and  $n_{CM}$  is the actual number of samples which fail before receiving PM acts,  $T_{Oi}$  is the actual operation time for sample  $i$ .  $T_{Oi}$  equals the maintenance schedule if sample  $i$  receives a PM act, and equals its lifetime if sample  $i$  fails in the end.

Figure 3 ~ Figure 10 demonstrate the performance of the PdM policy using 4 USCs. Based on the results, the best performance of the PdM policy using 3 USCs for  $CCR/CPR=2$  comes close to 0.02CPR, while the reliability based USC gives slightly worse results. Based on the results in Figure 3 ~ Figure 10, the best performance of the PdM policy using a certain USC and a given noise level of degradation process may differ greatly from its suboptimal performance. For example,  $r_A=0.020$  CPR for the time-based USC for  $CCR/CPR=2$  and  $n=3$ , and  $r_A=0.033$  CPR for the time-based USC for  $CCR/CPR=2$  and  $n=20$ . This verifies the importance of optimizing the USC for a PdM policy.

In practice, the cross validation method can be adopted to obtain the optimal value for a given USC based on historical samples. Actually, these historical samples are not only a prerequisite for optimizing a USC, they are used to establish the model of degradation processes for a PdM policy as well.

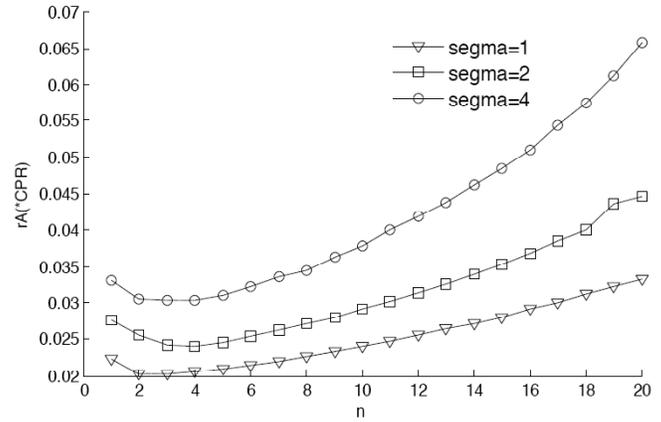


Figure 3:  $r_A$  with respect to different  $n$  for time-based updating stopping condition for  $CCR/CPR=2$ .

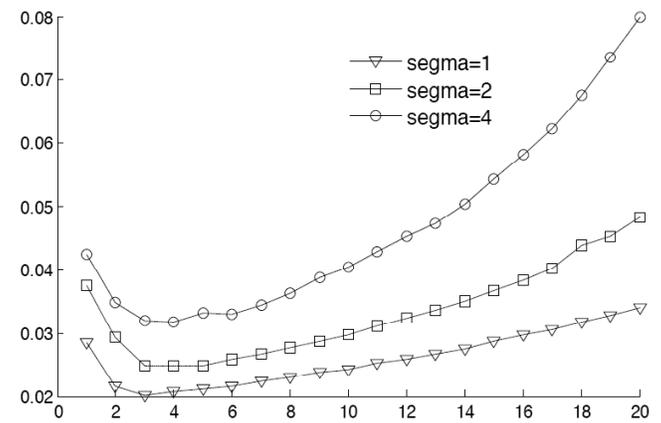


Figure 4:  $r_A$  with respect to different  $n$  for time-based updating stopping condition for  $CCR/CPR=4$ .

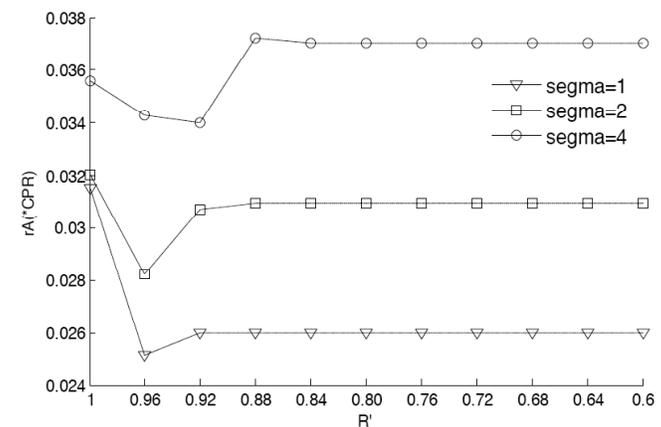


Figure 5:  $r_A$  with respect to different reliability threshold for reliability-based updating stopping condition for  $CCR/CPR=2$ .

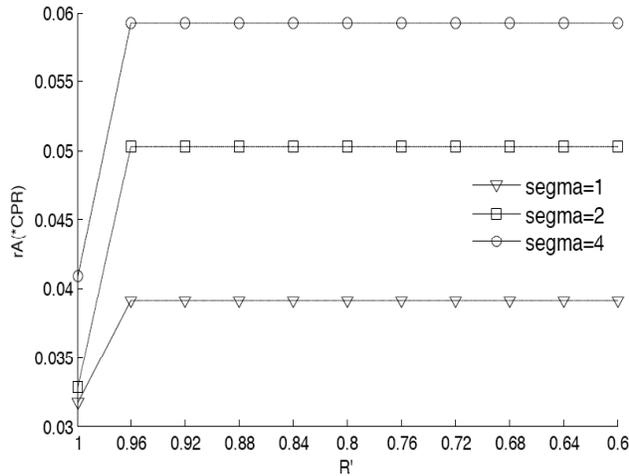


Figure 6:  $r_A$  with respect to different reliability threshold for reliability-based updating stopping condition for CCR/CPR=4.

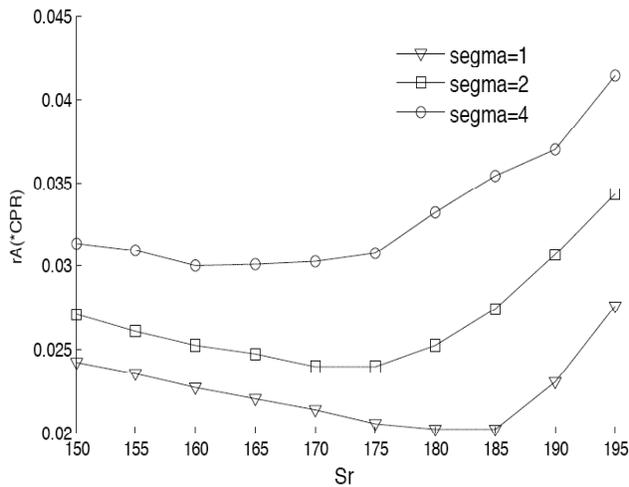


Figure 7:  $r_A$  with respect to different residual life threshold for condition-based updating stopping condition for CCR/CPR=2.

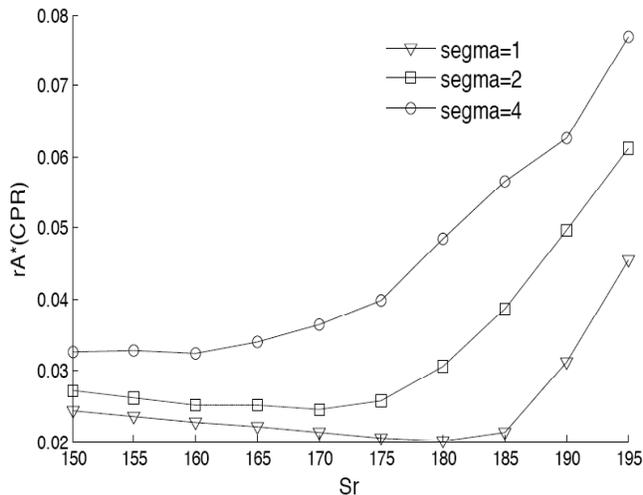


Figure 8:  $r_A$  with respect to different residual life threshold for condition-based updating stopping condition for CCR/CPR=4.

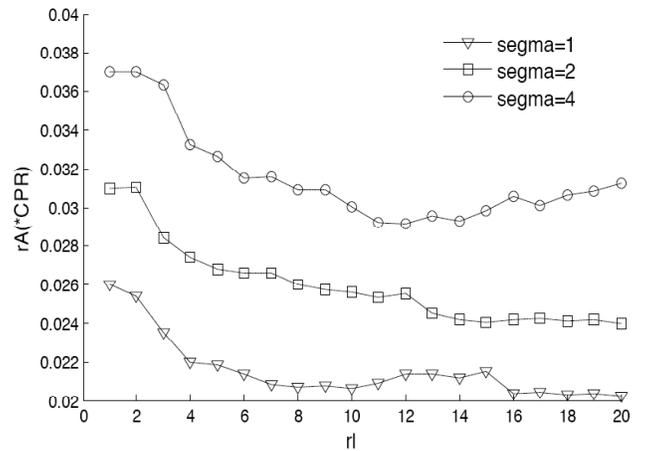


Figure 9:  $r_A$  with respect to different residual life threshold for residual life-based updating stopping condition for CCR/CPR=2.

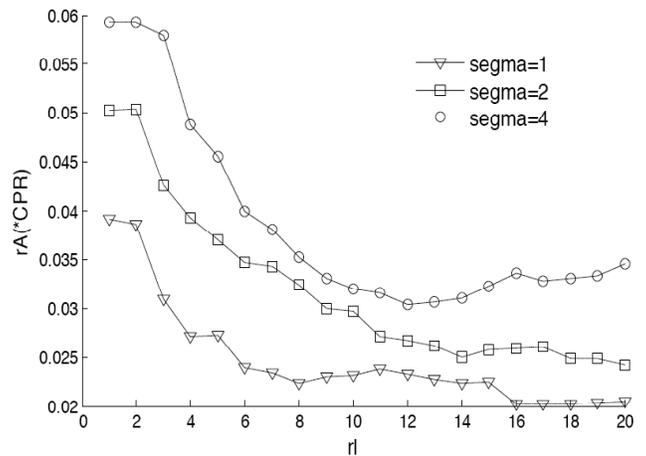


Figure 10:  $r_A$  with respect to different residual life threshold for residual life-based updating stopping condition for CCR/CPR=4.

5. CONCLUSION

This paper reviews the common USCs, proposes a residual life based USC, and evaluates the influence of the USCs on the effectiveness of a PdM policy. The commonly used USCs are concretely defined in a PdM policy based on the stochastic linear degradation model. An extensive numerical investigation compares the performances of the PdM policy using different USCs. The investigation results verify the importance of optimizing the USC for a PdM policy. Investigating the effect of a USC on the effectiveness of a system-level PdM policy may be a valuable topic in the future.

REFERENCES

[1] Zio E, Compare M. Evaluating maintenance policies by quantitative modeling and analysis. Reliability Engineering and System Safety 2013; 109: 53-65. <http://dx.doi.org/10.1016/j.res.2012.08.002>

- [2] Jardine AKS, Lin DM, Banjevic D. A review on machinery diagnostics and prognostics implementing condition-based maintenance. *Mechanical Systems and Signal Processing* 2006; 20(7): 1483-1510.  
<http://dx.doi.org/10.1016/j.ymsp.2005.09.012>
- [3] Liao H, Elsayed EA, Chan LY. Maintenance of continuously monitored degradation systems. *European Journal of Operational Research* 2006; 175(2): 821-835.  
<http://dx.doi.org/10.1016/j.ejor.2005.05.017>
- [4] Camci F. System maintenance scheduling with prognostics information using genetic algorithm. *IEEE Transactions on Reliability* 2009; 58(3): 539-552.  
<http://dx.doi.org/10.1109/TR.2009.2026818>
- [5] You MY, Li L, Meng G, Ni J. Cost-effective updated sequential predictive maintenance policy for continuously monitored degrading systems. *IEEE Transactions on Automation Science and Engineering* 2010; 7(2): 257-265.  
<http://dx.doi.org/10.1109/TASE.2009.2019964>
- [6] Kaiser KA, Gebraeel NZ. Predictive maintenance management using sensor-based degradation models. *IEEE Transactions on Systems, Man, and Cybernetics, Part A: Systems and Humans* 2009; 39(4): 840-849.  
<http://dx.doi.org/10.1109/TSMCA.2009.2016429>
- [7] Curcuru G, Galante G, Lombardo A. A predictive maintenance policy with imperfect monitoring. *Reliability Engineering and System Safety* 2010; 95: 989-997.  
<http://dx.doi.org/10.1016/j.ress.2010.04.010>
- [8] Gebraeel NZ. Sensory-updated residual life distributions for components with exponential degradation patterns. *IEEE Transactions on Automation Science and Engineering* 2006; 3(4): 382-393.  
<http://dx.doi.org/10.1109/TASE.2006.876609>
- [9] Meeker WQ, Escobar LA. *Statistical Methods for Reliability Data*. New York: Wiley 1998.
- [10] Kim YS, Kolarik WJ. Real-time conditional reliability prediction from on-line tool performance data. *International Journal of Production Research* 1992; 30(8): 1831-1844.  
<http://dx.doi.org/10.1080/00207549208948125>
- [11] Gebraeel NZ, Lawley MA, Li R, *et al.* Residual-life distributions from component degradation signals: a Bayesian approach. *IIE Transactions* 2005; 37(6): 543-557.  
<http://dx.doi.org/10.1080/07408170590929018>

---

Received on 14-10-2014

Accepted on 25-10-2014

Published on 19-11-2014

DOI: <http://dx.doi.org/10.15377/2409-9848.2014.01.01.2>

© 2014 Ming-Yi You; Avanti Publishers.

This is an open access article licensed under the terms of the Creative Commons Attribution Non-Commercial License (<http://creativecommons.org/licenses/by-nc/3.0/>) which permits unrestricted, non-commercial use, distribution and reproduction in any medium, provided the work is properly cited.