Solar Azimuth Angle in the Tropical Zone

Soulayman Shaher-Soulayman

Higher Institute for Applied Sciences and Technology, Damascus, Syria

Abstract: Basing on the concept of sun position, appropriate relationships for determining the solar azimuth angle were derived. The derived formulas were compared with those available in literature and proposed to determine the solar azimuth angle all over the year and all over world. Some comments were given on some of them. Here it was found that the formulation of Braun and Mitchell [1], recommended by Duffie and Beckman [2] in their respectful book could not be applied in the tropical zone. The calculated results using the derived formulas were compared with the available results in literature.

Keywords: Solar azimuth angle, tropical zone, formulations applicability, comparison.

1. INTRODUCTION

Azimuth is generally defined as the angle in the horizontal plane measured from the meridian's north (or south) to the location of the line in question. There are several types of meridians in use including true or geodetic, astronomic, magnetic, grid, record, assumed, etc. The true or geodetic meridian is the line that passes through a mean position between the earth's geographic north and south poles. The astronomic meridian is obtained through astronomic observations of the Sun or North Star (Polaris) and it is one instance of the true or geodetic meridian. The astronomic meridian's location is a function of the direction of gravity and the axis of rotation of the Earth and it determined from a mathematical approximation of the Earth's shape [3].

The knowledge of solar azimuth angle γ_s is important in observing the daily apparent Sun position trajectory. The azimuth angle is the compass direction from which the sunlight is coming. Outside the tropical zone and at solar noon, the sun is always directly south in the northern hemisphere and directly north in the southern hemisphere. The azimuth angle varies throughout the day. At the equinoxes, the sun rises directly east and sets directly west regardless of the latitude, thus making the azimuth angles -90° at sunrise and 90° at sunset. In general however, the azimuth angle varies with the latitude and time of year and a self-consistent and suitable procedure to calculate the sun's position throughout the day is required.

It is of the greatest importance in solar energy systems design, to be able to calculate the solar altitude and azimuth angles at any time for any location on the Earth. This can be done using the following three angles latitude φ , hour angle ω and declination δ . The latitude angle φ is one of the main angles that relate the position of the observer on the Earth's surface and the Earth center. The angle between a line, drawn from a point on the Earth's surface to the center of the Earth, and the Earth's equatorial plane, is φ . The intersection of the equatorial plane with the surface of the Earth forms the equator and is designated as 0 degrees latitude. The Earth's axis of rotation intersects the Earth's surface at 90° latitude (North Pole) and -90° latitude (South Pole). Any location on the surface of the Earth then can be defined by the intersection of a longitude angle and a latitude angle.

Several trigonometric functions $(\sin \gamma_s, \cos \gamma_s \text{ and }$ $\tan \gamma_s$) were proposed for determining the solar azimuth angle γ_s with respect to either south or north directions. However, it seems that there is a disadvantage in using tangential function as in 90° or in 270° angles γ_s reaches infinity. This discontinuity can cause uncertain calculation errors in the general simulation of fluent changes due to disruptions especially in extreme cases when the whole range $0-360^{\circ}$ or $0-2\pi$ has to be modeled for the sun paths fluently. Therefore, both cosine and sine functions were used in the ASHREA handbook [4] and by Tregenza and Sharples [5] while the cosine functions were recommended by lqbal [6], Kittler and Darula [7] and Muneer [8] especially for computer algorithms. The sine functions were used by Duffie and Beckman [2] while tangent functions were used by Buckner [9], Ali [10] and Duffie and Beckman [2]. Thus misunderstandings can be caused in calculations for equinox days at globe poles where zero azimuths for the whole days would be indicated. More sophisticated calculation methods for accurate solar positions were suggested by Blanc and Wald [11] and

^{*}Address correspondence to this author at the Higher Institute for Applied Sciences and Technology, Damascus, Syria; Tel: +963999700803; Fax: +963115134199; E-mail: soulayman1952@gmail.com

recommended for astronomy purposes and satellite data evaluations but their use has to expect rather tedious and complex algorithms.

The present work is intended to derive some of the used formulas and to verify the applicability of different methods in the tropical zone.

2. SUN POSITION

If the origin of a set of coordinates is defined at the Earth's center C (see Figure 1), the *y* axis with a unit vector of **j** can be a line from the origin intersecting the equator at the point where the meridian of the observer at *Q* crosses. The *x*-axis with a unit vector of **i** is perpendicular to the *y*-axis and is also in the equatorial plane. The third orthogonal axis *z* with a unit vector of **k** may then be aligned with the Earth's axis of rotation. Then the unit direction vector *r* pointing to the Sun may be described in terms of its direction cosines s'_i , s'_j and s'_k relative to the *x*, *y*, and *z* axes, respectively using hour ω and declination δ angles.

$$s' = s'_i i + s'_i j + s'_k k$$
⁽¹⁾

$$s'_{i} = \cos(\delta) \times \sin(\omega); \ s'_{j} = \cos(\delta) \times \cos(\omega);$$

$$s'_{k} = \sin(\delta)$$
(2)

When the Sun is observed from an arbitrary position on the Earth's surface, it is important to determine the Sun position relative to a coordinate system based at the point of observation, not at the center of the Earth. The conventional Earth-surface based coordinates are a vertical line (straight up) and a horizontal plane containing a north-south line and an east-west line. The position of the Sun relative to these coordinates can be described by two angles: the solar altitude angle α and the solar zenith angle γ_s . A unit direction vector **s** pointing toward the Sun from the observer location *Q* could be defined.

$$s = s_i \mathbf{i} + s_j \mathbf{j} + s_k \mathbf{k} \tag{3}$$

where, **i**, **j**, and **k** are unit vectors along the east, *E*, south, *S*, and vertical, *Z* axes respectively. The direction cosines of **s** relative to the *E*, *S*, and *Z* axes are s_E , s_S and s_Z , respectively. These may be written in terms of solar altitude α and azimuth γ_s as:



Figure 1: Sun position vector in the Earth's center and Earth's surface coordinate systems.

$$s_i = \cos(\alpha) \times \sin(\gamma_s); s_j = \cos(\alpha) \times \cos(\gamma_s);$$

$$s_k = \sin(\alpha)$$
(4)

These two sets of coordinates are interrelated by a rotation about the *E* axis through the latitude angle and translation along the Earth radius *QC*. We will neglect the translation along the Earth's radius since this is about 1/23,525 of the distance from the Earth to the Sun, and thus the difference between the direction vectors **s** and **s'** is negligible. Note that this rotation about the *E* axis is in the negative sense based on the right-hand rule. Accordingly, for the azimuth angle sign convention used in this text, that is, that the solar azimuth angle is measured from due south in a clockwise direction (see Figure (1)), the following relations could be obtained:

$$s_{j} = s'_{i}; s_{j} = s'_{j} \times \sin(\varphi) - s'_{k} \times \cos(\varphi);$$

$$s_{k} = s_{i} \times \cos(\varphi) + s_{k} \times \sin(\varphi)$$
(5)

Substituting Eq. (2) and Eq. (4) into Eq. (5) for the direction cosine gives:

$$sin(\alpha) = cos(\theta_z) = sin(\delta) \times sin(\varphi) + cos(\delta) \times cos(\varphi) \times cos(\omega)$$
(6)

 $\sin(\gamma_s) = \cos(\delta) \times \sin(\omega) / \cos(\alpha)$ (7)

$$\cos(\gamma_s) = [-\sin(\delta) \times \cos(\varphi) + \cos(\delta) \\ \times \sin(\varphi) \times \cos(\omega)] / \cos(\alpha)$$
(8)

Here, it should be mentioned that, Sproul [12] used vector analysis in deriving similar three formulas for determining the solar azimuth angle but with regard to the north direction. For a common direction, the equations (6) to (8) are totally equivalent to those of Sproul [12].

The ASHREA Handbook [4] is using both Eqs. (7) and (8). Iqbal [6] noted that Eq. (7) gives improper values when $\gamma_s \ge 90^\circ$ and mentioned that it should be avoided. Therefore he recommended the use of the following equation with relation to the south orientation:

$$\cos(\gamma_s) = (\sin\alpha \times \sin\varphi - \sin\delta) / (\cos\alpha \times \cos\varphi)$$
(9)

A similar equation is proposed by Kittler and Darula [7] but with regard to the north orientation:

$$\cos(\gamma_s) = (\sin\delta - \sin\alpha \times \sin\varphi) / (\cos\alpha \times \cos\varphi)$$
(10)

The tangent form can be derived applying Eqs. (7) and (8) as:

$$\tan(\gamma_s) = \frac{\sin(\gamma_s)}{\cos(\varphi_s)} = \frac{\sin(\omega)}{\cos(\varphi)} - \frac{\sin(\varphi)}{\cos(\omega)}$$
(11)
$$[-\tan(\delta) \times \cos(\varphi) + \sin(\varphi) \times \cos(\omega)]$$

Even Eq. (11) is derived for the azimuth of the sun measured clockwise from astronomic south, it is the same equation used by Buckner [9] for the azimuth of the sun measured clockwise from astronomic north. Therefore, additional information should be provided when using Eq. (11) depending on the reference direction. On the other hand, the determination of solar azimuth angle, with regard to both reference directions, requires the determination of accurate time (hour angle ω), latitude φ and declination δ angles.

3. SOLAR AZIMUTH ANGLE CALCULATION PROCEDURES

Equations (7) and (8) allow to determine the azimuth angle with a good approximation and to answer to question: In which quadrant the sun will be in any instant during the day? Here it should be mentioned that, angles should be interpreted with care because the inverse sine, i.e. $x = sin^{-1}(y)$ or $x = sin^{-1}(y)$ arcsin(y), has multiple solutions, only one of which will be correct. In calculating γ_s from the equations (7) to (11) a problem occurs whenever the absolute value of $\gamma_{\rm s}$ is greater than 90° because of the computational devices. However, for calculating the solar azimuth angle some authors use only one of these equations with applying some conditions additionally. In the northern hemisphere's tropical zone, the sun rays come from the direction of the North Pole (NP) at the solar noon of summer solstice while they come from the direction of the South Pole (SP) at winter solstice's solar noon in the southern hemisphere's tropical zone. The reverse is observed in the southern hemisphere Therefore, some arguments should be (SH). considered exclusively:

- 1) There is a period in the year where the solar rays at noon come from the orientation of the equator while during the rest period of the year they come from the orientation of the poles. Therefore, at solar noon, γ_s takes the value of 0° or 180° depending on the day number in the year and its inclusion in one of these two periods.
- 2) At the equator there is two periods, the length of each is a half year. During the first one, from 22/9 to 21/3, the solar rays come from the South Pole's direction, while they come from the direction of the North Pole during the other period, from 22/3 to 21/9. Therefore, at solar

noon, γ_s takes, with respect to the south direction, the value of 0° during the first period and 180° during the second period.

3) During the equinox, the sun passes from east to west at the equator. Therefore, γ_s takes the value of -90° before solar noon and 90° after it with respect to the south direction.

The solution of Eqs. (7), (8), (9) and (11) should satisfy the above mentioned arguments.

3.1. Braun and Mitchell Formulation [1]

Duffie and Beckman [2] recommended the formulation of Braun and Mitchell [1]. According to this formulation, the solar azimuth angle is:

$$azimuth = C_1 \times C_2 \times \gamma_s + 180 \times C_3 \times (1 - C_1 \times C_2)/2$$
(12)

where γs is determined by Eq. (7) or Eq. (11),

$$C_{1} = 1 \text{ if } abs(\omega) \leq \arccos\left[\frac{\tan(\delta)}{\tan(\varphi)}\right] \text{ or } -1$$

$$if \ abs(\omega) > \arccos\left[\frac{\tan(\delta)}{\tan(\varphi)}\right]$$
(13)

$$C_2 = 1 \text{ if } (\varphi - \delta) \ge 0 \text{ or } -1 \text{ if } (\varphi - \delta) < 0;$$

$$C_3 = 1 \text{ if } \omega \ge 0 \text{ or } -1 \text{ if } \omega < 0$$
(14)

The absolute value of $tan(\delta)/tan(\phi)$ in the tropical zone is greater than one for several days in the year (the winter and summer solstices are within these days). So, the value of $arccos[tan(\delta)/tan(\phi)]$ (see Eq. (13)) can not be definitive. Therefore, the formulation of Braun and Mitchell [1] is not applicable in the tropical zone.

3.2. Cosine Function Methods

When calculating the solar azimuth angle using Eqs. (8) and (9) it was found that they are of the same accuracy and the give the same results. Thus, it is sufficient to use the results of one of them. On the other hand, as $\cos(-x) = \cos(x)$ and these equations lead to the same values of solar azimuth angle for negative and positive hour angles in the northern hemisphere (NH) during the period from 22/9 to 21/3 and from 22/3 to 21/9 in the southern hemisphere (SH), where $|\gamma_s| \leq 90^\circ$. During this period the sunshine duration is less or equal 12 hours and the values of $\gamma_{\rm s}$ could be found in the first and fourth quarters only. Therefore, the correct results could be obtained by multiplying these equations by sign(ω). Thus,

$$\gamma_{s} = \arccos\{[-\sin(\delta) \times \cos(\varphi) + \cos(\delta) \times \sin(\varphi) \times \cos(\omega)] / \cos(\alpha)\} sign(\omega)$$
(8a)

$$\gamma_{s} = \arccos[(\sin\alpha \times \sin\varphi - \sin\delta) / \cos\alpha \times \cos\varphi] sign(\omega)$$
(9a)

Here, it should be mentioned that, Duffie and Beckman [13] recommended the use of the following equation:

$$\gamma_{s} = \left| \arccos\left[\frac{(\sin\alpha \times \sin\varphi - \sin\delta)}{(\cos\alpha \times \cos\varphi)} \right] \right| sign(\omega)$$
(15)

The results of (15) are absolutely identical to those of the equation (9a). During the period, where the sunshine duration is less or equal 12 hours, $\gamma_{\rm s}$ =0° should be considered at solar noon according to second argument. Moreover, during the other half year (from 22/3 to 21/9 in northern hemisphere and from 22/9 to 21/3 in the southern hemisphere) there is a period where, at solar noon the sun rays come from the opposite direction (pole direction) and the maximal absolute value of γ_s is greater than 90°. During this period the daylight length is greater than 12 hours and the values of γ_s could be found in the four quarters and additional conditions should be applied, when using the cosine functions, to make the calculated results correct. At the second and third guarters, these values could be calculated as:

$$\gamma_{s} = \begin{cases} 360^{\circ} + \arccos\left[\frac{(-\sin\delta\cos\varphi + \cos\delta\sin\varphi\cos\omega)}{\cos\alpha}\right] \\ sign(\omega), for \ \omega \le 0^{\circ} \\ \arccos\left[\frac{(-\sin\delta\cos\varphi + \cos\delta\sin\varphi\cos\omega)}{\cos\alpha}\right], \\ for \ \omega > 0^{\circ} \end{cases} \end{cases}$$
(8b)

$$\gamma_{s} = \begin{cases} 360^{\circ} + \arccos\left[\frac{(\sin\alpha \times \sin\varphi - \sin\delta)}{\cos\alpha}\right] sign(\omega), \\ for \ \omega \le 0^{\circ} \\ \arccos\left[\frac{(-\sin\delta\cos\varphi + \cos\delta\sin\varphi\cos\omega)}{\cos\varphi\cos\alpha}\right], for \ \omega > 0^{\circ} \end{cases}$$
(9b)

At first and fourth quadrants, the solar azimuth angle could be calculated using the following equations:

$$\gamma_{s} = 180^{\circ} - \arccos\left[\frac{(-\sin\delta\cos\varphi + \cos\delta\sin\varphi\cos\omega)}{\cos\alpha}\right]sign(\omega)$$
(8c)

According to the above mentioned, during the period from 22/3 to 21/9 in the tropical zone of NH, the sunshine duration of each studied day should be divided into three period: one at the morning just after sunrise, where Eq. (8c) could be applied; one at afternoon just before sunset where Eq. (8c) could be applied and the third is between these two periods where Eq. (8b) could be applied. Kittler and Darula [7] proposed the following equation for calculating the solar azimuth angle with regard to the north orientation:



They mentioned that, Eq. (16) is applicable all over the year.

3.3. Sine and Tangent Functions Methods

During the half year, from 22/9 to 21/3, in the NH, Eqs. (7) and (11) could be applied as they are for calculating the solar azimuth angle. The results of these two equations are identical during this period while they differ in sign during the other half year. During the half year, from 22/3 to 21/9, in the NH, the values of γ_s could be found in the four quarters and additional conditions should be applied before applying each of the Eqs. (7) and (11). For calculating γ_s at the second and third quarters using these equations, they should be modified as:

$$\gamma_s = 180^\circ - \arcsin[\cos(\delta) \times \sin(\omega) / \cos(\alpha)]$$
 (7a)

 $\gamma_s = 180^\circ - \arctan\{\sin(\omega) / \\ [-\tan(\delta) \times \cos(\varphi) + \sin(\varphi) \times \cos(\omega)]\}$ (11a)

In order to estimate the true azimuth of a line in a study area in central Orlando, Florida, United States, Ali [10] collected and used six sets of sun observation for azimuth data; three with telescope direct and three reverse, including horizontal circle's readings and time. In this study, Ali [10] used a Gauss-Markov model to represent the errors in the true azimuth estimation process. For interpreting the measured solar azimuth

angle, Eq. (11), with reference to the astronomic north direction, was used Ali [10]. The hour angle, according to Ali [10], varies from 0° (at solar noon) to 360° (at solar noon). The obtained values were then normalized from 0° to 360° by adding algebraically a correction as follows. When $0^{\circ} \le \omega \le 180^{\circ}$ a correction of 180° should added to γ_{s} if γ_{s} is positive otherwise the correction should be 360°. If $180^{\circ} < \omega \le 360^{\circ}$ a correction of 0° should added to γ_{s} if γ_{s} is positive otherwise the correction should be 180°.

4. RESULTS AND DISCUSSIONS

In order to verify the applicability of the proposed equations in the tropical zone, the solar azimuth angle as a function of solar hour angle is traced for several latitudes in the equinox day 21st March as well as in the extreme solstices, i.e. 21st June and 21st December in that zone. The obtained results were also compared with those of available procedures in the literature.

4.1. Cosine Function Methods

Winter solstice and equinox are located in the half year, from 22/9 to 21/3 in the NH. During this period the sunshine duration is less or equal 12 hrs. Therefore, Eq. (8a) could be applied for calculating solar azimuth angle during 21st December and 21st March. The calculated results for the latitudes: $\varphi = 0^{\circ}$ (the Equator), $\varphi = 10^{\circ}N$, $\varphi = 15^{\circ}N$ and $\varphi = 20^{\circ}N$ are given in the Figures 2 and 3.

The observed discontinuity, in the results of Equator at equinox (see Figure **3**), is not real because at solar noon the direction of the sun vector position changes but its projection value is zero.

Summer solstice (21st June) is located in the half year, from 22/3 to 21/9 in the NH. During this period the sunshine duration is greater than 12 hrs. Moreover, at solar noon the sun shines from the side of the North Pole. Therefore, the projection of solar vector will be located at four quadrants. The sun rises from the south east direction and sets at the south west direction, while at the rest of the day the projection of solar vector will be located at second and third quadrants. Therefore, it is preferable to calculate...

Therefore, Eqs (8b) and (8c) could be applied for calculating solar azimuth angle during 21^{st} June in NH. In order to visualize the continuity it is recommended to start at the reference direction and reserve the clockwise direction. Thus the values of 0° and 360° are



Figure 2: Solar azimuth angle on 21st December for (equator, \blacksquare), (φ =10°N, \blacklozenge), (φ =15°N, \blacktriangle) and (φ =20°N, x).



Figure 3: Solar azimuth angle on 21^{st} March for (equator, \aleph), ($\varphi = 10^{\circ}N$, •), ($\varphi = 15^{\circ}N$, +) and ($\varphi = 20^{\circ}N$, -).

on the reference direction. The calculated results for the latitudes: $\varphi = 0^{\circ}$ (equator), $\varphi = 10^{\circ}N$, $\varphi = 15^{\circ}N$ and $\varphi = 20^{\circ}N$ are given in the Figure 4. It is seen from Figure 4 that, the above mentioned three periods are clearly demonstrated. The length of boundary periods increases with the increase of latitude value. Moreover, some kind of discontinuity appears in the γ_s hour angle dependence when the sun passes through the eastwest axis (see Figure 4). This discontinuity doesn't have any physical interpretation and could not be eliminated by changing the reference direction or by playing on the correction factor. Therefore, it is quite understood why Kittler and Darula [7] applied the proposed equations by them with the correction factor



Figure 4: Solar azimuth angle on 21^{st} June for (equator, \blacktriangle), ($\varphi = 10^{\circ}N$, x), ($\varphi = 15^{\circ}N$, \varkappa) and ($\varphi = 20^{\circ}N$, \bullet).

for determining the solar azimuth angle in the Equator where the discontinuity doesn't appear, see Figure **4**. However, during the period from 22/3 to 21/9 in the NH and during the period from 22/9 to 21/3 in the SH, a serious problem appears in the solar azimuth angle calculations.

4.2. Sine Function Methods

Eq. (7) could be applied for calculating solar azimuth angle during 21^{st} December and 21^{st} March. The calculated results are identical to those given in the Figures **2** and **3**. When applying Eq. (7a) for

calculating the solar azimuth angle during the period from 22/3 to 21/9 at NH the results for the summer solstice at latitude φ =20°N are given in the Figure 5. There are two points that are not marked in the Figure 5: a) One is of the value of 270° in the observed discontinuity in the Figure 5 and b) One is of the value of 90° in the observed discontinuity in the Figure 5. Thus real discontinuities were not available. Apart from this comment, the results of Figure 5 are identical to those of Figure 4.

The results of tangent function are identical to the above obtained results.



Figure 5: Solar azimuth angle on 21^{st} June for ($\varphi = 20^{\circ}N$).

4. CONCLUSIONS

Basing on the sun position vector components several equations were found for determining the solar azimuth angle in the tropical zone. When applying each of these formulae by itself using an additional correction factor, it was found:

The formulation of Braun and Mitchell [1], recommended by Duffie and Beckman [2] could not be applied in the tropical zone.

The derived formulae in this work are effective in determining the solar azimuth angle during a half a year where the maximal possible sunshine duration is \leq 12 hrs.

During the other half year, an attention should be paid to the interval of time when the sun position vector projection passes through the east-west axis.

At that moment, some kind of discontinuity appears on the dependence of solar azimuth angle on the solar hour angle, but this discontinuity is not real as for the south direction -180° and 180° denote the same point.

The behavior of the vector position projection confirms that there is not any discontinuity. This will be the matter for a separate publication.

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