An Algorithm for Determining Optimum Tilt Angle and Orientation for Solar Collectors

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Abstract: Proper installation, especially the tilt angle, directly affects the system's output. Determination of the optimal tilt angle of a solar cell module depends on the solar radiation characteristics, season, and reflectivity in the local area. One of the important parameters that affect the performance of a solar collector is its tilt angle with the horizon. This is because of the variation of tilt angle changes the amount of solar radiation reaching the collector surface. A mathematical model is presented for determining the optimum tilt angle at different orientations (surface azimuth angle) for the solar collector, on a daily basis, as well as for a specific period such as monthly, seasonally and yearly basis. The optimum angle was computed by searching for the values for which the extraterrestrial solar radiation on the collector surface is a maximum for a particular day or a specific period. The results reveal that changing the tilt angle 12 times in a year (i.e. using the monthly optimum tilt angle) or even 4 times/year (i.e. using the seasonally optimum tilt angle) maintains approximately the total amount of solar radiation near the maximum value that is found by changing the tilt angle daily to its optimum value. This achieves a yearly gain in solar radiation of 5.5% at 0^o latitude to 78% at 60^o latitude more than the case of a solar collector fixed on a horizontal surface.

Keywords: Optimum tilt angle, tilt factor, mathematical model.

1. INTRODUCTION

Solar systems, like any other system, need to be operated with the maximum possible performance. This can be achieved by proper design, construction, installation, and orientation. Nevertheless, the performance of a solar collector is highly influenced by its orientation (with respect to the Equator) and its angle of tilt with the horizon (with respect to the ground). The orientation of the collector is described by its azimuth G and tilt B angles. This is due to the fact that both the azimuth and tilt angle change the solar radiation reaching the surface of the collector. Fixed photovoltaic panels and solar thermal heating systems are widely used in most commercial applications. It is therefore important to find the right angle to maximize the harnessing of solar energy.

In the previous studies, it was concluded that in the northern hemisphere, the optimum orientation is south facing and the optimum tilt angle depends only on the latitude. No definite value is accepted by all researchers for the optimum tilt angle. Therefore, several attempts were made to determine, or at least to estimate, optimum tilt angle B_{opt} theoretically and experimentally. For example, Hottel [1] suggested B_{opt} =L+20°, Taybout and Löf [2] proposed B_{opt} =L+(10→20°), Heywood [3] concluded that B_{opt} = L − 10º, Löf and Taybout [4] proposed B_{opt} =L+(10→30°), Yellott [5] reported B_{opt} = L ± 20º, Kern and Harris [6] suggested L+10°, Lunde [7] and Garge [8] suggested B_{opt} = L ± 15º, Lewis [9] reported B_{opt} = L ± 8º, where L is latitude of the location and where plus and minus signs are used in winter and summer, respectively. Theoretical models for B_{opt} were suggested by Lewis [9], who considered two different models for B_{opt} while El-Kassaby [10] and El-Kassaby & Hassab [11] introduced an analytical equation to get the daily optimum angle at any latitude. They also concluded that the optimum tilt angle for any period could be obtained by integrating the analytical equation over the required period. Soulayman [12] by correcting the proposed method of El-Kassaby [10] presented an analytical method for predicting solar collector optimum tilt angle of any orientation. Shariah *et al.* [13] showed that the optimum inclination angle for the maximum solar fraction is about L+(0→10°) for the northern region (represented by Amman) and about L+(0→20°) for the southern region (represented by the town of Aqaba). These values are greater than those for maximum solar radiation (which is commonly used as an indicator) at the top of the collector by about 5 to 8°. Skeiker [14] presented his study aiming to develop an analytical procedure for calculating optimum tilt angle for any period of time. Kaldellis *et al*. [15] provided theoretical investigation that based on the validation of the assumption that the optimum angle for such applications coincides with the angle that provides maximum exploitation of solar potential during winter months. The common approach used by researchers has been to calculate the tilt angle which maximizes the amount of solar radiation received by the collector. Finally Diaz *et al.* [16] used a mathematical model is

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used to estimate the daily extraterrestrial radiation received by a unit area in order to generate the optimal PV tilt angles in the Philippines. In the past years, computer programs have been used and the results have shown that the optimum yearly tilt angle is almost equal to the latitude (see for example [17, 18].

Skeiker [14] presented his study aiming to develop an analytical procedure to obtain formulae which require the least number of parameters to determine B_{opt} for any chosen day, latitude in either hemisphere and for any value of G. This goal was not achieved by him because: a) His procedure is essentially the same procedure of El-Kassaby [10], and b) His effort to compute the optimum tilt angle for the main Syrian zones is based on a fatal error in his main formulae. So, the aim of the present work is to correct Skeiker [14], El-Kassaby [10] and Diaz *et al.* [16] results and some of Skeiker's statements regarding available literature. Moreover, as Skeiker [14] aimed to determine B_{opt} for any chosen day, latitude in either hemisphere and for any value of G, but as he failed in giving any value for B_{opt} at G ≠ 0 the present work helps him in achieving his aim correctly.

2. METHODOLOGY

The value of the optimum tilt angle is affected by different factors such as: a) the type of application, i.e. stand alone or grid connected; b) maximization the amount of collectable radiation for the whole year or a certain period of time; c) actual climatic condition of the site, regarding snow fall, dust storms or polluted air. Therefore, the main part of previous works in this field can be characterized as follows: 1) Many of the available in literature approaches are too approximate and they are mostly for specific locations while a part is based on the assumption that solar sunrise and sunset on solar collector are tilt independent (see for example [10, 14]); 2) None of the analytical methods, except [17], can be used when G≠ 0^o; 3) Different atmospheric transmittance models are not taken into account; 4) The analytical method presented in [19] can be applied only in the heating seasons, i.e. from 21 September to 21 March in the Northern Hemisphere, and from 21 March to 21 September in the Southern Hemisphere. Moreover, the model suggested in [9] was concerned with a narrow range rang of L (between 30^o and 35^o); 5) The use of some analytical methods [6,19] one must know several meteorological parameters such as monthly average daily hemispherical radiation; monthly average daily beam radiation, and monthly average daily diffuse radiation on a horizontal surface; 6) The use of some analytical methods is very complicated (in [20] for example); 7) Skeiker [14] stated that only four articles, namely [6,19-21], give an analytical treatment with focusing on El-Kassaby [10,11] articles and he mentioned that previous studies do not recommend any definite value of the tilt angle and this statement is not true (see for example [10, 12]). Oloketuyi S Idowu, Oyewola M Olarenwaju and Odesola I Ifedayo [22] optimized the collection of solar energy within the period of its availability in order to increase its utilization and also to enhance performance of heating systems that depend on it through appropriate determination of optimum solar collector tilt angles. It was mentioned in [22] that the optimum tilt angles for solar heating for periodic tracking of the sun in the region within latitudes 1° and 14° were predicted as L + 25° for November, December and January; L + 15° for February, September and October; L − 15° for August; L − 25° for May, June and July; and L for March and April. The results of this work confirmed that solar radiation on tilted surface increases with latitude.

As Skeiker [14] based his calculations on extraterrestrial solar radiation intercepted on a surface the proposed methodology in this work will be based on extraterrestrial solar radiation also. So, the daily extraterrestrial solar radiation intercepted on a surface tilted by an angle B to the horizontal plane and oriented by an angle G from equator direction at latitude L can be expressed as follows [12]:

$$H(N,L,B,G) = \left(12 \times \frac{3600\,Gsc}{\pi}\right) C(N) \times \{A2[\sin(Wss) - \sin(Wsr)] + A1(Wss - Wsr) - A3[\cos(Wss) - \cos(Wsr)]\} \frac{J}{m^2} \quad (1)$$

where G_{sc} = 1367 W/m^2 is the solar constant, N is the number of the day, in consideration, in the year starting from 1 January, W_{ss} and W_{sr} are the sunset and sunrise hour angles (in rad) respectively:

$$Wss = \min\left\{\arccos[-\tan(\delta)\tan(L), \arccos\left(-\frac{A1}{A4}\right) + \arcsin\left(\frac{A3}{A4}\right)\right\} \quad (2)$$

$$Wsr = \max\left\{-\arccos[-\tan(\delta)\tan(L)], -\arccos\left(-\frac{A1}{A4}\right) + \arcsin\left(\frac{A3}{A4}\right)\right\} \quad (3)$$

$$C(N) = 1 + 0.034\cos\left(\frac{2\pi N}{365}\right) \quad (4)$$

$$\delta = -23.45\cos\left[\frac{2\pi(N + 10.5)}{365}\right] \quad (5)$$

$$A1 = \sin(\delta)[\sin(L)\cos(B) - \sin(B)COS(L)\cos(G)] \quad (6)$$

$$A2 = \cos(\delta)[\cos(L)\cos(B) + \sin(B)\sin(L)\cos(G)] \quad (7)$$

$$A3 = \cos(\delta)\sin(B)\sin(G) \qquad A4 = (A2^2 + A3^2)^{1/2} \quad (8)$$

2.1. South Facing Case

For a south facing surface Skeiker [14] mentioned "Referring to Eq. (1), at a certain location on a particular day N, all the parameters are considered constant except B". Here Skeiker [14] made his first fatal error as W_{ss} is B dependent during the period started from March 21st to September 21st (see eqs (2) and (3)). $W_{ss} = -W_{sr}$ for horizontal and for a south facing surface only. Figure **1** demonstrates the tilt dependence of sunset solar hour angle on a south facing tilted surface for latitude L=60^o on two representative days (N=161 for the period started from 22/3 to 21/9 and N=354 for the period started from 22/9 to 21/3). The behavior of solar sunset hour angle dependence on surface tilt for any latitude in Northern Hemisphere is the same. The difference is restricted to values. It is seen from Figure **1** that for south facing collector solar sunset hour angle on a tilted surface equals solar sunset hour angle on a horizontal surface. This is due to the fact that sun can't rise on tilted surface before rising on the horizon.

The average daily total extraterrestrial solar radiation over a time period started from the first day number of the studied period N_1 to its last day number N_2 is:

$$H = \sum_{N=N1}^{N2} \frac{H(N,L,B,G)}{(N2 - N1 + 1)} \quad (9)$$

When differentiating (9) with respect to B and equating result to zero one can obtain an algebraic equation (10), optimum tilt B_{opt} is the solution of this equation.

$$\sum C(N)\left\{\begin{aligned} &\left(\frac{\partial A2}{\partial B}\right)[\sin(Wss) - \sin(Wsr)] \\ &+A2\left[\cos(Wss)\left(\frac{\partial Wss}{\partial B}\right) - \cos(Wsr)\left(\frac{\partial Wsr}{\partial B}\right)\right] \\ &+\left(\frac{\partial A1}{\partial B}\right)(Wss - Wsr) + A1\left(\frac{\partial Wss}{\partial B} - \frac{\partial Wsr}{\partial B}\right) \\ &-\left(\frac{\partial A3}{\partial B}\right)[\cos(Wss) - \cos(Wsr)] \\ &+A3\left[\sin(Wss)\left(\frac{\partial Wss}{\partial B}\right) - \sin(Wsr)\left(\frac{\partial Wsr}{\partial B}\right)\right] \end{aligned}\right\} = 0 \quad (10)$$

For south facing collector ($W_{ss} = -W_{sr}$ and G=0) the optimum tilt angle, at a particular day ($B_{opt,d}$), is the solution of the algebraic equation:

$$\sin(Wss)\left(\frac{\partial A2}{\partial B}\right) + A2\cos(Wss)\left(\frac{\partial wss}{\partial B}\right) + A1\left(\frac{\partial Wss}{\partial B}\right) + Wss\left(\frac{\partial A1}{\partial B}\right) = 0 \quad (11)$$

For south facing solar collector (G=0) $W_{ss} = -W_{sr}$, and W_{ss} is B independent only during the period started from September 21st to March 21st (see Figure **1**). The solution of equation (11) becomes:

$$Bopt.d = L - \arctan\left[\frac{Wss\tan(\delta)}{\sin(Wss)}\right] \quad (12)$$

Here Skeiker [14] made his second fatal error when mentioning that for a south facing collector, equation (12) is applicable all over the year. This fatal error is made by El-Kassaby [10,11] also.

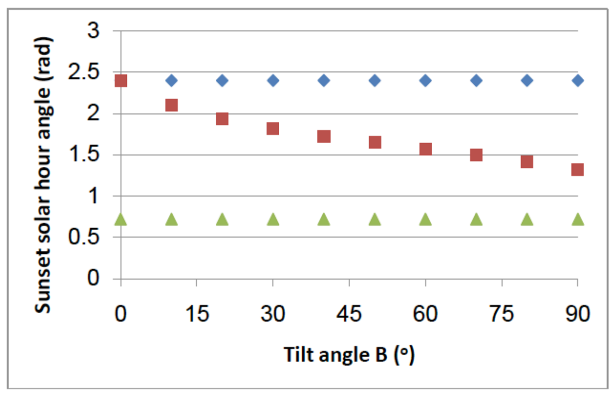

Figure 1: Solar sunset hour angle on horizontal surface (♦ on day number 161 and ▲ on day number 354) and on tilted surface (■ on day number 161 and ▲ on day number 354) at latitude of 60^o.

In some cases, it is not practical to design a solar collector for which the tilt angle changes every day. However, it may be possible to change it once a month. Therefore, the optimum monthly tilt can be obtained from equation (10). For south facing collector (G=0) $W_{ss} = -W_{sr}$ daily, the optimum tilt angle, at a particular month ($B_{opt,m}$), is the solution of the algebraic equation:

$$\sum C(N)\left\{\begin{aligned} &\left(\frac{\partial A2}{\partial B}\right)\sin(Wss) + A2\cos(Wss) \\ &\left(\frac{\partial Wss}{\partial B}\right) + Wss\left(\frac{\partial A1}{\partial B}\right) + A1\left(\frac{\partial Wss}{\partial B}\right) \end{aligned}\right\} = 0 \quad (13)$$

The solution of equation (13), for south facing solar collector (G=0), during the period started from

September 21st to March 21st, where W_{ss} is B independent becomes:

$$Bopt.m = L - \arctan\left[\sum_{N} C(N) Wss \sin(\delta) / \sum_{N} C(N)\cos(\delta)\sin(Wss)\right] \quad (14)$$

Here Skeiker [14] made his third fatal error when mentioning that for a south facing collector, equation (14) is applicable for all months all over the year and repeated his before mentioned uncertainties. Even Skeiker [14] mentioned that "the sunset hour angle W_{ss} varies with the day number N, as well as with the tilt angle B" before providing his formula regarding $B_{opt,m}$ he ignored this statement when deriving this formula.

Finally, Skeiker [14] mentioned "It is worth in this regard to mention that the monthly-averaged daily mean sunset hour angle for the tilted surface is given by:

$$Wss = \min\{\arccos[-\tan(\delta)\tan(L)], \arccos[-\tan(\delta)\tan(L-B)]\}$$

where "min" means the smaller of the two items in the bracket" and here Skeiker [14] demonstrates his misunderstanding of the treated question. In this regard, what is the value of δ which should be taken in the case of a period?

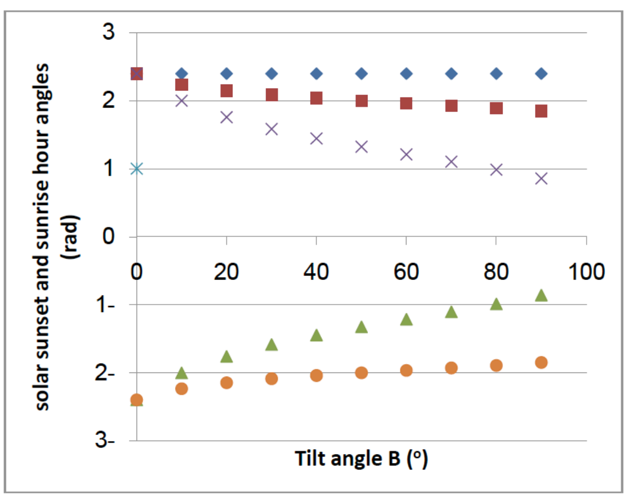

Figure 2: Day number 161 solar sunset (♦ - horizon, ■ – surface azimuth G=25° and x - surface azimuth G=-25°) and sunrise (▲ – surface azimuth G=25° and ● - surface azimuth G=-25°) hour angles at latitude of 60°.

2.2. Non South Facing Case

For non south facing solar collector, the optimum tilt is the solution of the non-linear algebraic equation (10) for the desired period. In this case, generally, one of W_{ss} and W_{sr} is B dependent at least all over the year and $W_{ss} \neq - W_{sr}$. This question is essential. Figures **2-4** show the dependence of W_{ss} and W_{sr} on surface tilt angle on 161 day number and G= ±25° for latitudes L=60°, L=30° and L=0° respectively.

It is seen from Figures **2-4** that, for non south facing surfaces, the analytical treatment becomes more complicated and this will be hopefully considered in the near future.

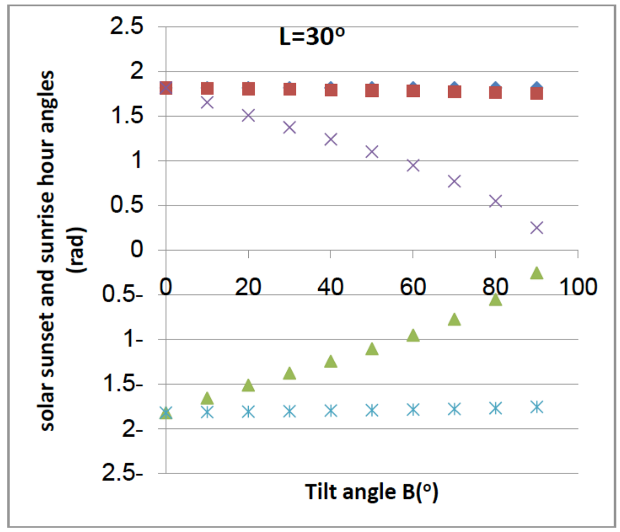

Figure 3: Day number 161 solar sunset (♦ - horizon, ■ – surface azimuth G=25° and x - surface azimuth G=-25°) and sunrise (▲ – surface azimuth G=25° and ж - surface azimuth G=-25°) hour angles at latitude of 30°.

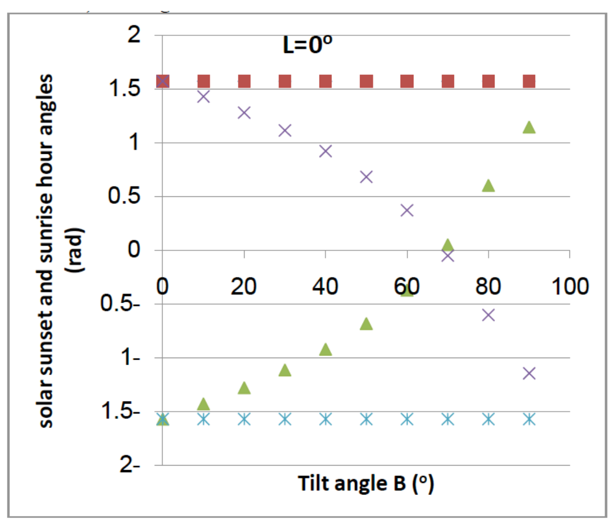

Figure 4: Day number 161 solar sunset (♦ - horizon, ■ – surface azimuth G=25° and x - surface azimuth G=-25°) and sunrise (▲ – surface azimuth G=25° and ж - surface azimuth G=-25°) hour angles at latitude of 0°.

As buildings roofs are not, in general, oriented in suitable way for solar thermal system collector installations. Thus, it is required to determine the optimum tilt for non-zero azimuthally oriented collectors. So, the question that may arise, is that is it worth changing the tilt angle once a month/season/year for a definite value of surface azimuth angle?

To answer this question, a computer program was devised, using Eq. (1)–(10), from which the optimum tilt angle for each day, month, season, year, surface azimuth angle as well as the total extraterrestrial solar radiation intercepted on 1m^2 of such a surface can be obtained.

3. RESULTS AND DISCUSSION

3.1. Daily Optimum Tilt Angle

In some practical applications, it is important to know the daily optimum tilt angle. The temporal variation of the movement of the sun in the sky over

Table 1: Daily Optimum Tilt Angle $B_{opt,d}$ (^{o}) for Different Latitudes

Latitude L in degrees (^{o})							Date	Day number (N)
60	50	40	30	20	10	0		
85.0	77.1	68.7	60.1	51.3	42.5	33.7	1/1	1
83.6	75.5	67.0	58.2	49.3	40.3	31.4	15/1	15
80.7	72.2	63.3	54.2	45.1	35.9	26.6	31/1	31
80.4	71.9	63.0	53.9	44.7	35.5	26.3	1/2	32
76.5	67.5	58.2	48.8	39.3	29.8	20.2	15/2	46
71.5	62.0	52.4	42.7	32.9	23.1	13.3	28/2	59
71.1	61.6	51.9	42.1	32.4	22.8	12.8	1/3	60
64.1	54.1	44.2	34.2	24.2	14.2	4.3	15/3	74
54.1	44.1	34.1	24.1	14.1	4.1	0	31/3	90
53.5	43.5	33.5	23.4	13.4	3.5	0	1/4	91
34.9	34.9	24.9	14.8	4.8	0	0	15/4	105
36.0	26.0	16.1	6.0	0	0	0	30/4	120
35.5	25.5	16.0	5.5	0	0	0	1/5	121
28.2	18.1	8.0	0	0	0	0	15/5	135
21.0	11.0	1.0	0	0	0	0	31/5	151
20.6	10.6	0.6	0	0	0	0	1/6	152
16.9	6.9	0	0	0	0	0	15/6	166
17.4	7.4	0	0	0	0	0	30/6	181
17.6	7.6	0	0	0	0	0	1/7	182
22.1	12.1	2.1	0	0	0	0	15/7	196
29.6	19.7	9.6	0	0	0	0	31/7	212
30.1	20.2	10.1	0	0	0	0	1/8	213
37.7	27.8	17.8	7.7	0	0	0	15/8	227
47.3	37.3	27.3	17.3	7.3	0	0	31/8	243
47.9	37.9	27.9	17.9	7.9	0	0	1/9	244
56.7	46.7	36.7	26.7	16.7	6.7	0	15/9	258
65.7	55.8	45.9	36.0	26.0	16.1	6.2	30/9	273
66.3	56.4	46.5	36.6	26.6	16.7	6.8	1/10	274
72.8	63.4	53.8	44.2	34.5	24.7	15.2	15/10	288
78.3	69.5	60.4	51.2	41.8	32.4	23.1	31/10	304
78.6	69.8	60.8	51.5	42.2	32.8	23.5	1/11	305
82.0	73.6	64.9	55.9	46.9	37.8	28.7	15/11	319
84.2	76.2	67.8	59.0	50.2	41.3	32.4	30/11	334
84.3	76.3	67.9	59.2	50. 3	41.4	32.6	1/12	335
85.3	77.4	69.1	60.4	51.7	42.9	34.1	15/12	349
85.1	77.2	68.8	60.2	51.4	42.6	33.8	31/12	365

any location during a day is considered by computing the tilt angle. The results obtained for different latitude angles are tabulated in Table **1**.

The results reveal that $B_{opt,d}$ increases as the latitude angle of the considered location increases, and varies with the day number N.

In order to compare our results with those available in literature, namely [12] and [14] as these methods are described as analytical and data is complete in these two references, $B_{opt,d}$ is calculated for day number 181 which could be considered as representative day for the period started from 22/3 to 21/9. The results are given on Figure **5**. Moreover, $B_{opt,d}$ is calculated using the mentioned three methods for the latitudes of 50° and 60° during the period started from 22/3 to 21/9. These two latitudes were chosen as no negative value for $B_{opt,d}$ is observed in [14]. As no differences between the results of the mentioned three methods during the period started from 22/9 to 21/3 the comparison is restricted to the period started from 22/3 to 21/9. The results are shown on Figures **6** and **7**.

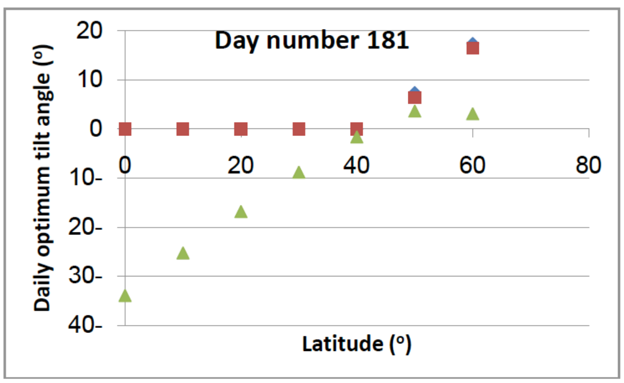

Figure 5: Latitude dependence of $B_{opt,d}$ calculated using our method (♦),that of [12] (■) and that of [14] (▲) during the day number 181.

It is seen from Figure **5** that there is a good agreement between the results of the method developed in this work and that of [12]. As mentioned before, no need to compare the results of the used methods, namely: i) the method presented in this work; ii) Skeiker method [14] or El-Kassaby [10,11] method; and iii) Soulayman [12] method, during the period started from 22/9 to 21/3 as these methods give the same results. It is seen from Figures **6** and **7** that there is a good agreement between the results of the method, developed in this work, and that of [12] while the method of [14] underestimates the optimum daily tilt angle during the period started from 22/3 to 21/9. The underestimation increases with the latitude angle increase.

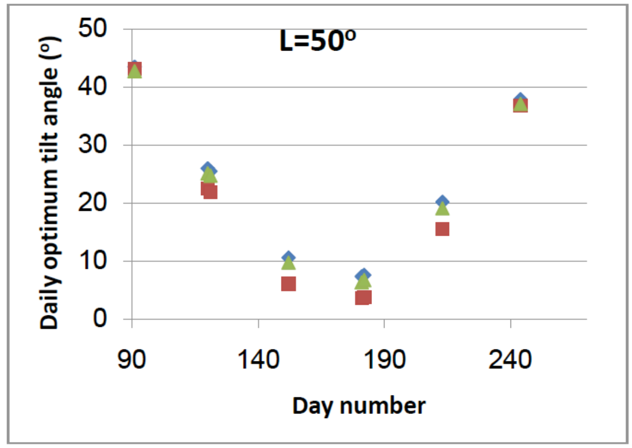

Figure 6: Calculated $B_{opt,d}$ using our method (♦),that of [12] (▲) and that of [14] (■) for 50° of latitude for period started from 21/3 to 21/9.

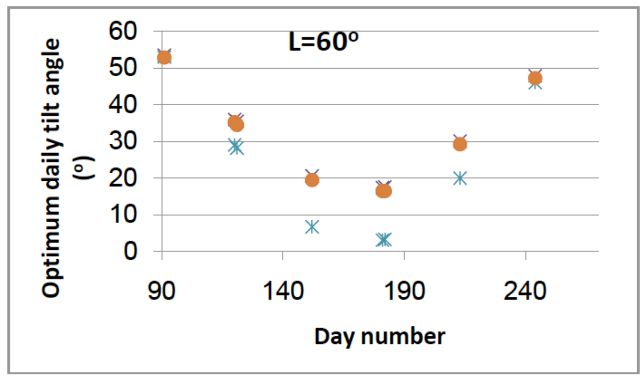

Figure 7: Calculated $B_{opt,d}$ using our method (●),that of [12] (x) and that of [14] (ж) for 60° of latitude for period started from 21/3 to 21/9.

3.2. Monthly Optimum Tilt Angle

The set of Eqs. (1)–(14) is used to obtain the monthly optimum tilt angle. Table **2** shows the results obtained for different latitude angles using different methods where "a" stands for this work, "b" stands for [14] and "c" stands for [12].

It is seen from Table **2** that no practical difference between the results of the mentioned three methods during the months January, February, March, September, October, November and December. This result is expected as for south facing collector sunset and sunrise hour angles are independent of tilt angle during the period started from 21/9 to 21/3. So, the comparison should be restricted to the period started from 21/3 to 21/9. Table **2** shows that the method of [14] underestimates the monthly optimum tilt angle. On the other hand, the results of the present work are in acceptable accordance with those of [12]. Moreover, it is found, see Table **2**, that the monthly optimum tilt

Table 2: The Monthly Optimum Tilt Angle $B_{opt,m}^{o}$ for Different Latitude Angles

Dec.	Nov.	Oct.	Sep.	Aug.	July	June	May	Apr.	Mar.	Feb.	Jan.	M	L°
33.8	28.6	15.4	0	0	0	0	0	0	3.6	20.3	30.8	a	
33.8	28. 6	15.4	-2.94	-20.2	-31.1	-33.8	-28.2	-14.8	3.6	20.3	30.8	b	0
33.9	28.9	15.9	1.5	0	0	0	0	0	4.6	20.3	30.8	c	
38.2	33.1	20.3	2.0	0	0	0	0	0	8.6	25.0	35.3	a	
38.2	33.1	20.3	2.03	-15.5	-26.7	-29.4	-23.7	-10.	8.6	25.1	35.3	b	5
38.2	33.1	20.3	2.8	0	0	0	0	0	8.4	25.1	35.3	c	
42.6	37.7	25.1	7.00	0	0	0	0	0	13.6	29.8	39.8	a	
42.6	37.7	25.1	7.00	-10.8	-22.3	-25.13	-19.17	-5.13	13.6	29.8	39.8	b	10
42.6	37.6	25.1	6.9	0	0	0	0	0	13.5	29.8	39.8	c	
47.0	42.2	29.9	12.0	0	0	0	0	0	18.5	34.6	44.3	a	
47.0	42.2	30.0	12.0	-6.05	-18.0	-20.9	-14.7	-0.3	18.5	34.6	44.3	b	15
47.0	42.2	29.9	11.8	0	0	0	0	0	18.4	34.6	44.3	c	
51.4	46.8	34.8	17.0	0	0	0	0	4.7	23.5	39.3	48.7	a	
51.4	46.8	34.8	17.0	-1.4	-13.7	-16.7	-10.3	4.5	23.5	39.3	48.8	b	20
51.4	46.7	34.8	16.7	0	0	0	0	2.6	23.3	39.3	48.8	c	
55.8	51.3	39.6	22.0	3.5	0	0	0	9.7	28.5	44.0	53.2	a	
55.8	51.3	39.6	21.9	3.3	-9.4	-12.6	-5.9	9.3	28.5	44.0	53.2	b	25
55.8	51.3	39.6	21.8	1.5	0	0	0	7.5	28.4	44.0	53.2	c	
60.2	55.8	44.4	27.0	8.5	0	0	0	14.7	33.5	48.8	57.7	a	
60.2	55.8	44.4	26.9	7.9	-5.4	-8.7	-1.7	14.1	33.4	48.8	57.7	b	30
60.2	55.8	44.4	26.7	6.4	0	0	0	12.8	33.3	48.8	57.7	c	
64.5	60.3	49.2	32.0	13.5	0	0	3.0	19.7	38.4	53.5	62.1	a	
64.5	60.3	49.2	31.9	12.4	-1.4	-4.9	2.4	18.9	38.4	53.5	62.1	b	35
64.5	60.3	49.2	31.6	11.3	0	0	0.8	17.6	38.2	53.5	62.1	c	
68.8	64.7	54.0	37.0	18.5	3.1	0	8.1	24.7	43.3	58.1	66.5	a	
68.8	64.7	54.0	36.8	16.9	2.3	-1.5	6.4	23.6	43.3	58.1	66.5	b	40
68.8	64.7	54.0	36.8	16.5	0.8	0	5.6	22.5	43.3	58.1	66.5	c	
73.1	69.1	58.8	42.0	23.5	8.1	2.7	13.1	29.7	48.3	62.8	70.8	a	
73.1	69.1	58.8	41.8	21.2	5.6	1.5	10.0	28.2	48.3	62.8	70.8	b	45
73.1	69.1	58.8	41.7	21.4	5.6	0	10.9	27.8	48.2	62.8	70.8	c	
77.2	73.5	63.5	46.9	28.5	13.1	7.7	18.1	34.7	53.3	67.4	75.1	a	
77.2	73.5	63.5	46.7	25.4	8.3	3.8	13.2	32.8	53.3	67.4	75.1	b	50
77.2	73.5	63.5	46.6	26.3	10.5	4.5	15.8	32.6	53.1	67.4	75.1	c	
81.2	77.7	68.2	52.0	33.5	18.1	13.3	23.1	39.7	58.2	71.9	79.2	a	
81.2	77.7	68.2	51.6	29.2	10.1	4.9	15.7	37.2	58.2	71.9	79.2	b	55
81.2	77.7	68.2	51.8	31.5	15.8	9.8	20.6	37.5	58.1	71.9	79.2	c	
85.1	81.7	72.8	57.0	38.5	23.1	18.3	28.1	44.7	63.1	76.3	83.2	a	
85.1	81.7	72.8	56.6	32.5	9.9	3.4	16.6	41.4	63.1	76.3	83.2	b	60
85.1	81.7	72.8	56.7	36.4	20.6	14.6	25.9	42.7	63.0	76.3	83.2	c	

"a" stands for this work, "b" stands for [14] and "c" stands for [12].

angle on March is higher than its value by about 6° (except L=0 where the difference is 3.6 °). Moreover, the half of the sum of monthly optimum tilt angles on March and September equals to the latitude angle with a good accuracy.

Nijegorodov *et al*. [21] reported that the dependence of the optimum tilt angle on temperature, pressure, humidity and visibility is very weak, but cloud cover has a significant effect. As the cloud cover increases, the optimum tilt angle is found to decrease. In the limiting case of complete cloud cover the optimum tilt angle is

Table 3: The Comparison between $B_{opt,dc}{}^{o}$ and $B_{opt,m}{}^{o}$

Latitude (°)							
60	**50**	**40**	**30**	**20**	**10**	**0**	**Day Number**
83.3 83.2	75.2 75.1	66.6 66.5	57.8 57.7	48.9 48.7	39.9 39.8	30.90 30.8	17
76.5 76.3	67.5 67.4	58.2 58.1	48.8 48.8	39.3 39.3	29.8 29.8	20.20 20.3	46
62.9 63.1	52.9 53.3	43.0 43.3	33.0 33.5	23.0 23.5	13.0 13.6	3.00 3.6	76
44.9 44.7	34.9 34.7	24.9 24.7	14.9 14.7	4.9 4.7	0 0	0 0	105
28.2 28.1	18.2 18.1	8.2 8.1	0 0	0 0	0 0	0 0	135
18.2 18.3	8.2 7.7	0 0	0 0	0 0	0 0	0 0	162
23.7 23.1	13.7 13.1	3.7 3.1	0 0	0 0	0 0	0 0	199
38.9 38.5	28.9 28.5	18.9 18.5	8.9 8.5	0 0	0 0	0 0	229
56.7 57.0	46.7 46.9	36.7 37.0	26.7 27.0	16.7 17.0	6.7 7.0	0 0	258
72.8 72.8	63.4 63.5	53.8 54.0	44.2 44.4	34.5 34.8	24.7 25.1	15.2 15.4	288
81.8 81.7	73.4 73.5	64.7 64.7	55.7 55.8	46.6 46.8	37.5 37.7	28.4 28.6	318
85.2 85.1	77.3 77.2	68.9 68.8	60.3 6.02	51.5 51.4	42.7 42.6	33.90 33.8	346

zero. This is true for any location, because when the beam (direct) solar radiation is zero, the only radiation to be intercepted is the diffuse radiation. The collector/absorber should, therefore, be installed horizontally for the global view of sky.

3.2.1. Monthly Optimum Tilt Angle Calculation on the Basis of Characteristic Days

Normally, regarding solar radiation calculations, one can represent the average daily monthly solar radiation by its value during characteristic days. Table **3** presents the day number of the characteristic days in the year and the optimum daily tilt angle during these days $B_{opt,dc}$ (upper line) and the corresponded $B_{opt,m}$ (lower line).

It is seen from Table **3** that the absolute difference between $B_{opt,dc}$ and $B_{opt,m}$ does not exceed 0.6°. So, it is reasonable to use this method for evaluating $B_{opt,m}$.

3.2.2. Monthly Optimum Tilt Angle Calculation Comparison

Nijegorodor *et al*. [21] presented a set of 12 equations (one for each month) for the estimation of monthly-averaged optimum tilt angle, $B_{opt,m}$ for any location that lies between latitude of 60° south to 60° north. These equations were used to compute the optimum tilt angle for latitudes of 40°, 50° and 60° for a full calendar year and a comparison of these angles with those obtained using the proposed method in this work, is given in Table **4** and plotted in Figure **8**. Results reveal that the equations of Nijegorodor *et al*. [21] give a little bit higher values of optimum angle for the month of April, May, and August as compared with that obtained in the present study while the agreement for other months is very good. So, a separate problem, concerning the equations of Nijegorodor *et al*. [21] corrections, arises. This question will be treated in the near future.

Here it should be mentioned that, due to the fact that the equations used in this work, as well as those used in [21], were optimized using mathematical techniques without taking into account the localized patterns of solar radiation falling over a particular location (region). As solar radiation is a locality dependent variable, therefore, it should be considered as one of the variables for optimization of any parameter that depends on it.

Table 4: Monthly Optimum Tilt Angle Values in Comparison with [21] Results

Month	L=40°		L=50°		L=60°	
	[21]	This work	[21]	This work	[21]	This work
Jan.	64.6	66.5	73.5	75.1	82.4	83.2
Feb.	55.8	58.1	65.5	67.4	75.2	76.3
March	44	43.3	54	53.3	64	63.1
April	30	24.7	40	34.7	50	44.7
May	13.2	8.1	22.5	18.1	31.8	28.1
June	0.8	0	9.5	7.7	18.2	8.1
July	5.6	3.1	14.5	13.1	23.4	23.1
Aug.	21.8	18.5	31.5	28.5	41.2	38.5
Sep.	38	37	48	46.9	58	57
Oct.	52	54	62	63.5	72	72.8
Nov.	62.2	64.7	71.5	73.5	80.8	81.7
Dec.	68.8	68.8	77.5	77.2	86.2	85.1

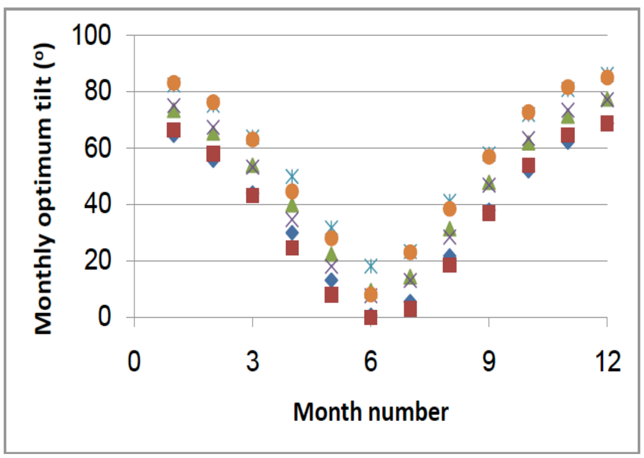

Figure 8: Calculated $B_{opt,m}$ using our method (■ stands for L=40°; x stands for L=50° and ● stands for L=60°) and that of [21] (♦ stands for L=40°; ▲ stands for L=50° and ж stands for L=60°).

The optimum tilt angle for the month of March and September is approximately equal to the latitude. For these months, a solar collector tilted at an angle equal to the latitude will receive solar radiation nearly normally. Similarly the optimum angle recorded for (20–22) March and September is approximately equal to the latitude of the location.

3.3. Seasonal and Yearly Optimum Tilt Angle

Since changing the tilt angle to its daily and monthly optimum values throughout the year does not seem to be practical, another possibility, such as changing the tilt angle once in a period of three months (i.e. seasonally), was considered. So, the seasonal optimum tilt angle $B_{opt,s}$ should be evaluated also. Here it should be mentioned that solar seasons are considered as follows. Solar winter is considered to cover the period started from 22/12 up to 21/3 while solar spring covers the period started from 22/3 up to 21/6. Solar summer covers the period started from 22/6 up to 21/9 and the solar autumn covers the period started from 22/9 up to 21/12. The results of $B_{opt,s}$ calculation are given in Table **5**.

The daily, monthly, and seasonal variation of optimum slope has been extended to evaluate the yearly optimum tilt angle, $B_{opt,y}$. The yearly optimum tilt angle is a fixed value for any solar collector throughout the course of a year. The optimum yearly tilt angles were evaluated for different latitude angles. The results are listed in Table **5**.

3.4. Tilt Factor

For purposes of solar process design and performance calculations, it is often necessary to calculate the hourly radiation on a tilted surface of a collector from measurements or estimates of solar radiation on a horizontal surface. The geometric tilt factor R_b, the ratio of beam radiation on the tilted surface to that on a horizontal surface at any time or period of time, can be calculated exactly in the case of extraterrestrial radiation by appropriate use of solar incidence angle on tilted surface and on horizon. For daily, monthly, seasonally and yearly values of R_b, the following equation can be used:

Table 5: The Seasonal and Yearly Optimum Tilt Angles for Different Latitudes

Latitude (°)													
60	55	50	45	40	35	30	25	20	15	10	5	0	
76.8	72.8	68.6	64.3	59.8	55.3	50.8	46.2	42.7	38.4	33.8	29.0	24.2	Win
35.7	30.7	25.7	20.7	15.7	10.7	5.7	0.7	0	0	0	0	0	Spr
35.1	30.1	25.1	20.1	15.2	10.1	5.1	0.1	0	0	0	0	0	Sum
76.6	72.7	68.5	64.1	59.7	55.2	50.6	46.0	42.5	38.2	33.5	28.8	24.0	Aut
55.1	51.3	47.2	42.8	38.3	33.7	29.0	24.4	19.7	14.9	10.2	5.4	0.7	year

$$Rb = \sum H(N, L, B, G) / \sum H(N, L, B = 0, G) \tag{15}$$

where no summation by N is used for obtaining daily values while the summation by N should cover the length of period in consideration.

When calculating the total yearly extraterrestrial solar radiation at B = 0, B = B_{opt} on a daily as well as on a monthly basis and taking the ratio between the value on a tilted surface to the value on a horizontal one for the same period of time, the corresponded tilt factor is calculated for each case. In the case of yearly evaluation of tilt, the results are presented in Table **6** and plotted in Figure **9**.

Table **6** shows that the total extraterrestrial solar radiation over the year received by a collector using $B_{opt,d}$ on a daily basis is almost the same as that received by a collector using $B_{opt,m}$ on a monthly basis and $B_{opt,s}$ on a seasonally basis. This means that it is worth designing a collector for which the tilt angle changes once a month or even once a season. Moreover, the amount of solar radiation on the tilted surface is comparable with the horizontal surface, i.e. Bopt,y = 0 for the latitude 0°. There is an overall increase of about 1.5% of solar radiation collected by the solar collector/absorber throughout the period of a year for the latitude of 10.0°. Moreover, an overall increase of about 3–65% are recorded for the latitudes from 15° to 60 °, respectively. The amount of solar radiation received at the surface of the fixed solar collector at annual optimum angle is 12% approximately lower than that recorded for the monthly optimum tilt angle. The yearly optimum tilt slope is relevant to big and heavy solar collectors that cannot be moved frequently.

4. CONCLUSIONS

Designing a solar collector in which the tilt angle can change monthly or even seasonally provides better efficiency. It is easy to notice that the optimum monthly

Table 6: Yearly Tilt Factor for Daily, Monthly, Seasonally and Yearly Optimum Tilt Angle

$B_{opt,y}$ (°)		$B_{opt,s}$ (°)		$B_{opt,m}$ (°)		$B_{opt,d}$(°)		B=0 °	L (°)
R_b	H_{ext}	Rb	H	R_b	H_{ext}	R_b	H_{ext}	H_{ext}	
1.000	3654	1.041	3806	1.055	3855	1.058	3865.7	3653.8	0
1.004	3653.2	1.063	3869	1.074	3908.5	1.076	3914.2	3637.6	5
1.015	3651.3	1.089	3920	1.098	3950.73	1.100	3956.6	3596.3	10
1.033	3647.9	1.120	3957	1.124	3970.17	1.129	3986.5	3530.2	15
1.058	3642.9	1.154	3971	1.160	3992.5	1.162	4000	3440	20
1.092	3635.6	1.188	3954	1.194	3974.52	1.196	3981.6	3326.7	25
1.136	3625.9	1.235	3943	1.243	3968.6	1.246	3976.0	3191.5	30
1.190	3612.8	1.293	3928	1.301	3952.28	1.305	3960.7	3035.9	35
1.256	3594.7	1.365	3908	1.372	3927.28	1.383	3935.6	2861.9	40
1.335	3569.2	1.450	3877	1.456	3891.5	1.459	3899.7	2672.1	45
1.430	3532.1	1.552	3834	1.555	3841.83	1.558	3849.9	2469.5	50
1.538	3474.8	1.667	3768	1.668	3768.86	1.672	3776.5	2258.8	55
1.650	3378.5	1.781	3647	1.783	3650.87	1.787	3658.5	2047	60

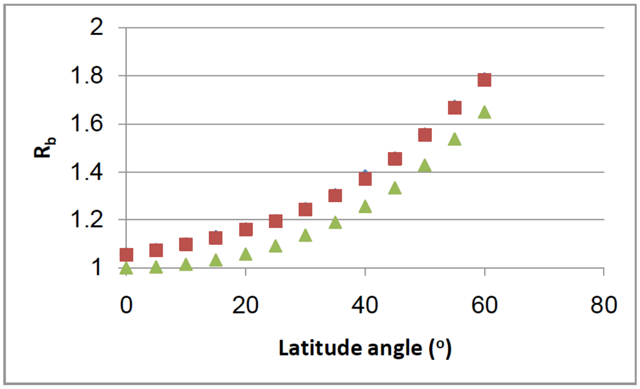

Figure 9: Latitude yearly R_b dependence calculated on the basis of daily (♦), monthly (■) and yearly (▲) optimum tilt angles.

tilt angle is the average daily optimum tilt angle during that month. Compared to the fixed slope for the whole year, seasonal adjustments of the solar collectors can receive from 4% to 78% more solar energy depending on latitude angle. This shows the importance of accurate slope angle and orientation. The position of the solar collectors can be easily adjusted when the supporting structure is designed accordingly. Once is designed, the adjustment of angles and orientations needs a very limited time and can be performed by an unqualified staff. The method described in this paper can be used to find these angles easily and accurately.

– The daily optimum tilt angle, measured from the horizontal, and orientation for a solar collector changes throughout the year can be determined by using the mathematical model described in the section on methodology.

– The daily, monthly, seasonally and yearly optimum tilt angles for a south facing solar collector changes throughout the year were determined for latitudes 0° -60°.

– Changing the tilt angle 12 times a year (i.e. using the monthly optimum tilt angle) or even 4 times a year (i.e. using the seasonally optimum tilt angle) maintains approximately the total amount of solar radiation near the maximum value that is found by changing the tilt angle daily to its optimum value. This achieves a yearly gain in solar radiation of 4% to 78% (depending on latitude angle) more than the case of a fixed solar collector at a horizontal surface.

– Even, the methods of El-Kassaby [10,11] and Skeiker [14] are not correct in calculating the optimum tilt angles during the period 22/3 to 21/9 they could be used with a high accuracy for latitudes lower than 40° in Northern Hemisphere in the condition of taking their negative values of optimum tilt as zero.

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