

# Systems of Measurement for the Construction of Geometrical Models

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**Abstract:** In the present work, the relevant *components* of the two basic system of measurement are defined through the analysis of existing systems of measurement for geometrical modeling. The major features of such systems related to the ways of formation of the two-dimensional complex drawing of a geometrical image on the basis of laws of projective connections are revealed. *Interrelations* of the elements of different systems of measurement with various projections of a geometrical image are defined. The *relative location of projections* of a geometrical image into the constructed two-dimensional complex drawings for various systems of measurement is discussed. *The rationale* behind a particular *arrangement of views* in the projective drawing of a product in the analyzed systems of measurement is explained.

**Keywords:** Geometry, systems of measurement, image, projection, law, projective connections, drawing, interrelations, arrangement of views.

## 1. PROBLEM STATEMENT

Various systems of measurement for geometrical modelling are used in the different countries [1-54]. In Australia, England, Holland, India, Japan, the United States of America (USA), and some other countries the *American system of an arrangement of projections* in engineering drawing is used. With respect to the front view, the top view is located above, the bottom view is located below, the left-side view - to the left, the right-side view - to the right of the front view, and the rear view - to the right of the product right-side view [1, 2, 41, 43, 45].

In Belarus, Germany, Kazakhstan, Kirgizstan, Poland, Russia, Ukraine and other countries the *European system of an arrangement of projections* in engineering drawing is used. Relative the front view, the top view is located below, the bottom view - above, the left-side view - to the right, the right-side view - to the left of the front view, and the rear view - to the right of the product left-side view [1-4, 6, 7].

The Indian Standard Institution (ISI) and the British Standard Institution (BSI) recommend the use of First Angle Projection method (the *European system*) now in all the institutions too [52,53].

As a rule, at various universities of the world one any system of an arrangement of projections in the drawing is taught as applied in design and technological works of the given country [8-13,15-27,40-51].

It may happen that an engineer developing a production process comes across a part drawing where the views' arrangement is different than he leans. As a result, he or she may not adequately understand the drawing, and thus part design including tolerances and requirements. As a consequence, an inadequate manufacturing process can be developed and/or a good part can be rejected on its inspection.

Realizable ways of formation of projections and views in the drawing are connected in consciousness of an engineer with a particular system of measurement for construction of geometrical models of a product [2, 4, 28-39].

The *American system of measurement* for the construction of a geometrical model of a product differs from the *European system of measurement*.

Thus, the essence of the problem (contradiction) consists in a necessity of adequate understanding the features of formation of projections in the drawing in used system of measurement (by everyone involved in design and product manufacture) and lack of the systematized knowledge of features of various ways of formation of projections in different systems of measurement in the geometrical modeling.

Therefore, the **purpose** of this work is *to reveal the features* of various systems of the measurement used in the geometrical modeling, and *to provide justifications for the arrangement of projections* in the product drawing.

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## 2. WORK TASKS

1. To analyze the existing systems of measurement used in the geometrical modeling defining their relevant *components*.

2. To reveal the *features* of the known systems the measurements connected with the ways of formation of the two-dimensional complex drawing of a geometrical image.

3. To define *interrelations* of elements of the discussed systems of measurement with different projections of a geometrical image.

4. To define *relative positioning of projections* of a geometrical image in the two-dimensional complex drawing for various systems of measurement.

5. To carry out a logic substantiation of an *arrangement of projections* in the projective drawing of a product.

## 3. THE MAIN PART

The *first mutual component* of the systems of measurement used for geometrical modeling is a set of *three mutually perpendicular planes* (Fig. 1).

In relation to the subject (to the student, the engineer, the researcher), one of these three planes is located horizontally ( $H-\Pi_1$ ) whereas other two planes ( $F-\Pi_2$ ,  $P-\Pi_3$ ) – vertically.

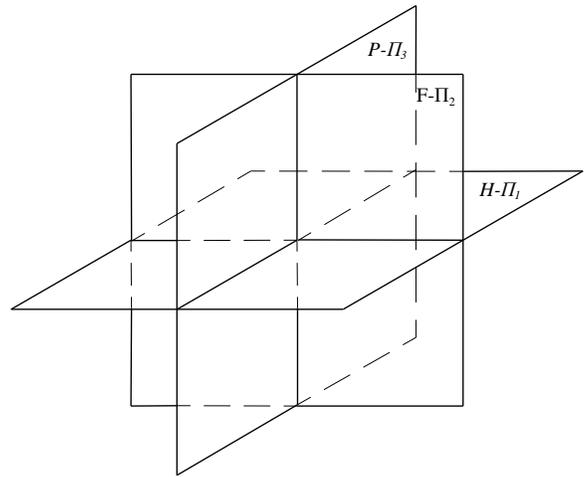
Horizontally focused plane  $H-\Pi_1$  is called as *the horizontal plane*. Vertically located plane  $F-\Pi_2$  is called as *the frontal plane*. Vertically focused plane  $P-\Pi_3$  is called as *the profile plane*.

For geometrical modeling, the method of orthogonal projections [2,4, and 44] is used.

In the method of orthogonal projections, horizontal plane  $H-\Pi_1$  is called as *the horizontal plane of projections*, frontal plane  $F-\Pi_2$  is called as *the frontal plane of projections*, and profile plane  $P-\Pi_3$  is called as *the profile plane of projections*.

The *second mutual component* of systems of measurement used for geometrical modeling is the *right three-dimensional system of coordinates OXYZ*, developed by Rene Descartes (1596-1650). The beginning of system of coordinates (a point O - Latin, origo) coincides with *the point of intersection* of all three planes.

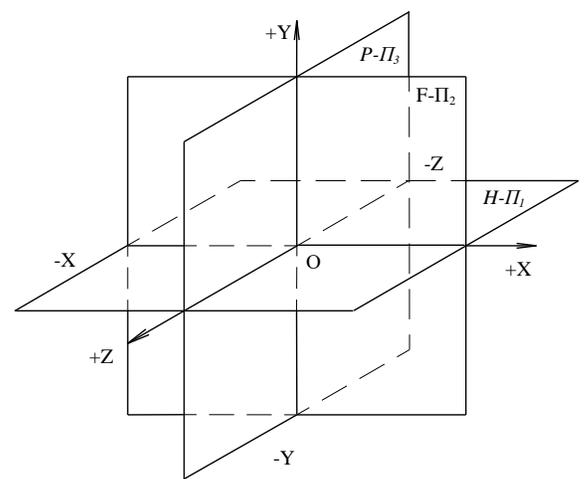
Thus, the *system of measurement* for geometrical modeling consists of three mutually perpendicular planes  $H-\Pi_1$ ,  $F-\Pi_2$ ,  $P-\Pi_3$  and connected with them right [5, 14] three-dimensional system of coordinates OXYZ.



**Figure 1:** Three mutually perpendicular planes of systems of measurement.

The allocation of a way of orientation of the right three-dimensional system of coordinates OXYZ relative to the planes of projections  $H-\Pi_1$ ,  $F-\Pi_2$ ,  $P-\Pi_3$  can be considered as the *first feature* of systems of measurement for geometrical modeling.

1. In Australia, England, Holland, Japan, the USA and a number of other countries, axes OZ and OX coincide with *horizontal plane*  $H-\Pi_1$ , axes OX and OY are located in *frontal plane*  $F-\Pi_2$ , and axes OY and OZ belong to *profile plane*  $P-\Pi_3$  (Fig. 2). All axes of coordinates coincide with the lines of intersection of the discussed planes [43, 45].



**Figure 2:** Linking of the system of coordinates OXYZ with planes of projections  $H-\Pi_1$ ,  $F-\Pi_2$ ,  $P-\Pi_3$  in the *American system of measurement*.

Because the positive direction of axis OZ relative to the origin O is directed *to the subject* (observer) for the right system of coordinates OXYZ, the positive coordinates of axis OX settle down *to the right* of the origin O and positive coordinates of axis OY are *above* the origin O.

Thus, axis OY is directed *upwards* concerning the origin O, axis OX is directed *to the right* concerning the origin O, and axis OZ is directed from the origin O *towards the observer* [45].

Accordingly, the portion of axis OX with negative coordinates is *to the left* of profile plane  $P-\Pi_3$ , the portion of axis OY with negative coordinates settles down *below* horizontal plane  $H-\Pi_1$ , the portion of axis OZ with negative coordinates is *behind* frontal plane  $F-\Pi_2$  [45].

The measurement system serves for placing and the description of geometrical images. *Geometrical images* are understood as abstract elements: a point, a line, a plane, a surface, a body [2-4, 6-27].

The *second feature* of systems of measurement for geometrical modeling is the *way of space partition* into semi spaces, quadrants, and octants.

For the description of the location of an object in a particular measurement, space is partitioned by a plane into two *semi spaces*, by two orthogonal planes into four *quadrants* (quarters), by three mutually perpendicular planes into eight *octants* (parts).

2. In the *American system* of measurement, space is partitioned into four *quadrants* by horizontal plane  $H-\Pi_1$  and *profile plane*  $P-\Pi_3$  (Fig. 3).

The *third feature* of systems of measurement for geometrical modeling is the *way of numbering* of the allocated parts of space (semi spaces, quadrants, and octants).

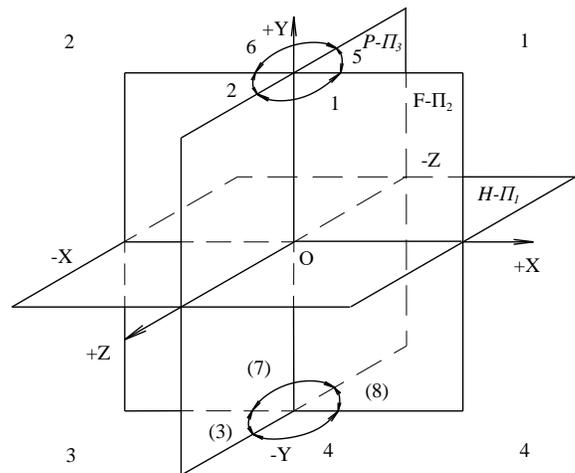
3. In the *American system* of measurement, numerical values of coordinates X and Y are positive for both axes — 1: +X, +Y. In the second quadrant, numerical values of coordinate Y are positive, and coordinates X are negative — 2: -X, +Y. In the third quadrant, numerical values of coordinates X and Y are negative for both axes — 3: -X, -Y. In the fourth quadrant, numerical values of coordinate Y are negative, and coordinates X are positive — 4: +X, -Y [45].

Quadrant number increase at consecutive viewing the quadrants from the positive direction +Z axes OZ

counter-clockwise (Fig. 3). Numbers 1, 2, 3, 4 quadrants are specified around the image.

The one eighth part of space, in which all numerical values of coordinates X, Y, Z for three axes OX, OY, OZ are positive +X, +Y, +Z is chosen as the first *octant*.

Numbers of the first four octants correspond to four numbers quadrants (quarters). The fifth, the sixth, the seventh and the eighth octants are bred behind the frontal plane of projections  $F-\Pi_2$  from the positive direction +Z axis OZ accordingly in the first, in the second, in the third and the fourth quadrant (Fig. 3).



**Figure 3:** Quadrants and octants numbering in the *American system of measurement*.

The revealed features of the American system of measurement differ from similar characteristics of the European system of measurement although there are some similarities.

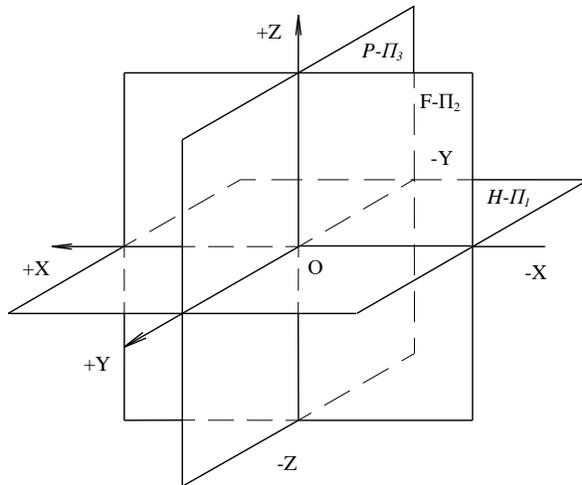
1. In the *European system of measurement*, the right [5, 14] the three-dimensional system of coordinates OXYZ is positioned differently relative to the planes of projections  $H-\Pi_1$ ,  $F-\Pi_2$ ,  $P-\Pi_3$  (Fig. 1) as shown in Fig. (4).

Axes OZ and OX coincide with *frontal plane*  $F-\Pi_2$ , axes OX and OY settle down in *horizontal plane*  $H-\Pi_1$ , and axes OY and OZ also belong to *profile plane*  $P-\Pi_3$  (Fig. 4). All the axes of coordinates are coincident with the intersection lines of the projection planes [4].

Because the positive direction of axis OZ with respect to the origin O is directed *upwards* for the right system of coordinates OXYZ, the positive coordinates of axis OX are *to the left* of the origin O and positive coordinates of axis OY settle down *before* frontal plane  $F-\Pi_2$  closer to the observer to the origin O (Fig. 4).

Thus, axis  $OY$  is directed from the origin  $O$  coordinates *towards the observer*, axis  $OX$  is directed *to the left* of the origin  $O$ , axis  $OZ$  is directed *upwards* relative to the origin  $O$ .

Accordingly, the portion of axis  $OX$  with negative coordinates is *to the right* of profile plane  $P-\Pi_3$ , the portion of axis  $OY$  with negative coordinates settles down *behind* frontal plane  $F-\Pi_2$ , the portion of axis  $OZ$  with negative coordinates is *below* horizontal plane  $H-\Pi_1$  [4].



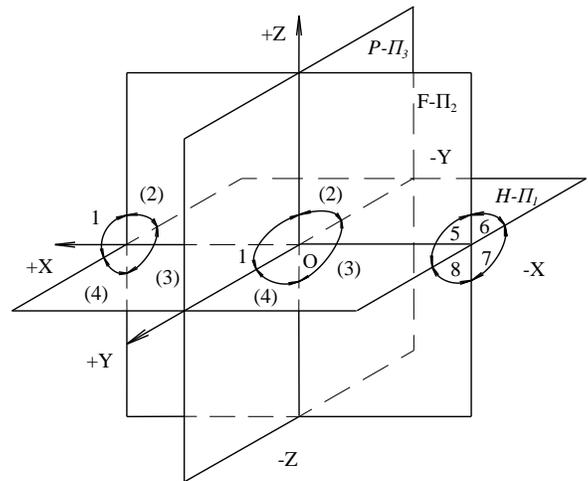
**Figure 4:** Linking of system of coordinates  $OXYZ$  with planes of projections  $H-\Pi_1$ ,  $F-\Pi_2$ ,  $P-\Pi_3$  in the *European system of measurement*.

2. In the *European system of measurement*, space is partitioned into four quadrants by horizontal plane  $H-\Pi_1$  and *frontal plane*  $F-\Pi_2$  (Fig. 5).

3. The way of numbering quadrants in the *European system of measurement* differs from the *American system of measurement*. In the first, *quadrant* numerical values of coordinates  $Z$  and  $Y$  are positive for both axes — 1:  $+Z$ ,  $+Y$ . In the second quadrant, numerical values of coordinate  $Z$  are positive, and coordinates  $Y$  are negative — 2:  $+Z$ ,  $-Y$ . In the third quadrant, numerical values of coordinates  $Z$  and  $Y$  are negative for both axes — 3:  $-Z$ ,  $-Y$ . In the fourth quadrant, numerical values of coordinate  $Z$  are negative, and coordinates  $Y$  are positive — 4:  $-Z$ ,  $+Y$  [4]. The quadrant number increases at consecutive viewing quadrants from the positive direction  $+X$  axes  $OX$  counter-clockwise (Fig. 5).

In the *European system of measurement*, the same as in the *American system of measurement*, the first *octant* is chosen as one eighth part of space, in which

all the numerical values of coordinates  $X$ ,  $Y$ ,  $Z$  for three axes  $OX$ ,  $OY$ ,  $OZ$  are positive  $+X$ ,  $+Y$ ,  $+Z$  is chosen.



**Figure 5:** Quadrants and octants numbering in the *European system of measurement*.

The difference for the first octant in the *European system* is that *horizontal plane*  $H-\Pi_1$  is located between axes  $OX$  and  $OY$  whereas in the *American system* *frontal plane*  $F-\Pi_2$  settles down, and *frontal plane*  $F-\Pi_2$  (Fig. 5) is located between axes  $OX$  and  $OZ$ , instead of horizontal plane  $H-\Pi_1$  (Figs. 2 and 3) in the *American system*.

Numbering of the first, the second, the third and the fourth octants, located *to the left* of profile plane  $P-\Pi_3$ , coincides with numbering of the first, the second, the third and the fourth quadrants the *European system* (Fig. 5).

The fifth, the sixth, the seventh and the eighth octants in the *European system* settles down *to the right* of profile plane  $P-\Pi_3$  accordingly in the first, the second, the third and the fourth quadrants (Fig. 5).

Let's consider construction of model of a geometrical image in the *American system of measurement*.

The *third octant* in the *third quadrant* is used in the *American system of measurement* for construction of model of a geometrical image (Fig. 3).

Therefore, segment  $AB$  of a straight line settles down in the third octant (Fig. 6).

The *method of rectangular (orthogonal) projection* is applied to construct the projections of segment  $AB$  [44].

The results of projection of segment  $AB$  into three mutually perpendicular planes  $F-\Pi_2$ ,  $H-\Pi_1$ ,  $P-\Pi_3$  are

frontal projection  $A_fB_f$ , horizontal projection  $A_hB_h$  and profile projection  $A_pB_p$  [45]. The constructed three-dimensional geometrical model of segment AB is shown at the left site in Fig. (6).

For the development of a two-dimensional complex drawing from a three-dimensional geometrical model, segment AB and projecting rays  $AA_f, AA_h, AA_p, BB_f, BB_h, BB_p$  are mentally removed. Horizontal plane  $H-\Pi_1$  with projection  $A_hB_h$  of the segment are rotated around the axis OX clockwise from the positive direction +X till its full coincidence with frontal plane  $F-\Pi_2$ . Profile plane  $P-\Pi_3$  with projection  $A_pB_p$  of the segment are rotated around the axis OY counter-clockwise from the positive direction +Y till its full coincidence with frontal plane  $F-\Pi_2$ . The constructed two-dimensional geometrical model of segment AB is shown on the right site in Fig. (6).

For the American system of measurement in the two-dimensional complex drawing of a geometrical image, the horizontal projection is located above the frontal projection, and the profile projection is located to the right of the frontal projection (Fig. 6).

Constructed frontal projection  $A_fB_f$ , horizontal projection  $A_hB_h$  and profile projection  $A_pB_p$  of segment AB satisfy the laws of projective connections 1-1, 2-2, 3-3 [4, 37].

Elements of the American system of measurement are unambiguously connected with the projections of a geometrical image.

1. Negative coinciding portions  $-X \equiv -Z$ ,  $-Y \equiv -Z$  of axes X and Y and axes of ordinates, applies OY, OZ

limit frontal plane  $F-\Pi_2$  and the frontal projection of a geometrical image.

2. Coinciding negative portions  $-X \equiv -Z$  of axes OX, OZ and coinciding positive portions  $+Y \equiv +Z$  of axes OY, OZ limit horizontal plane  $H-\Pi_1$  and the horizontal projection of a geometrical image.

3. Coinciding negative portions  $-Y \equiv -Z$  of axes OY, OZ and coinciding positive portions  $+X \equiv +Z$  of axes OX, OZ limit profile plane  $P-\Pi_3$  and the profile projection of a geometrical image.

For the realized way of construction of the complex drawing (Fig. 6) in the American system of measurement, the horizontal projection is the bottom view, and the profile projection is the left-side view.

Thus, if planes of projections are considered to be opaque in the projective drawing of a product then the bottom view settles down from above the front view and the left-side view settles down to the right of the front view. Conversely, in the countries using the American system of measurement, the bottom view settles down from below from the front view, and the left-side view settles down to the left of the front view. Conditions needed for realization of such accepted arrangement of views in the projective drawing of a product will be considered in the separate publication.

Let's consider the construction of model of a geometrical image in the European system of measurement. In the European system of measurement, the first octant in the first quadrant is used for construction of model of a geometrical image as shown in Fig. (5).

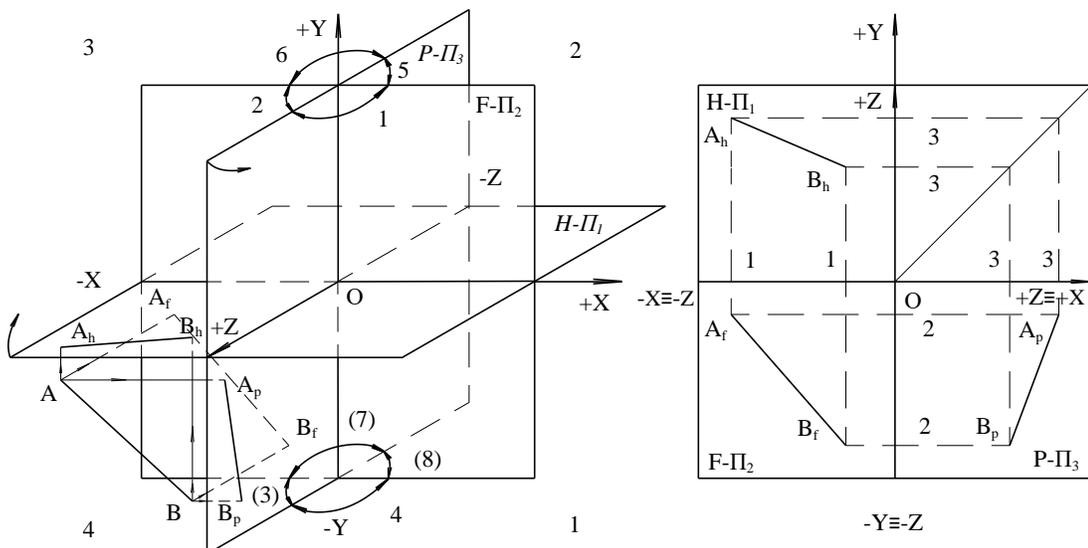


Figure 6: Geometrical models of segment AB of a straight line in the American system of measurement.

That is why segment AB of a straight line settles down in the first octant (Fig. 7).

The method of orthogonal projecting is also applied to develop projections of segment AB [44].

The results of projections of segment AB into three mutually perpendicular planes  $F-\Pi_2$ ,  $H-\Pi_1$ , and  $P-\Pi_3$  are the frontal projection  $A_2B_2$ , horizontal projection  $A_1B_1$  and profile projection  $A_3B_3$  [4]. The constructed three-dimensional geometrical model of segment AB is shown to the left in Fig. (7).

For construction of the two-dimensional complex drawing from three-dimensional geometrical model, segment AB and projecting rays  $AA_2$ ,  $AA_1$ ,  $AA_3$ ,  $BB_2$ ,  $BB_1$ ,  $BB_3$  are mentally removed.

Horizontal plane  $H-\Pi_1$  with projection  $A_1B_1$  of segment AB is rotated about the axis of abscissas OX clockwise from a positive direction +X till its full coincidence with frontal plane  $F-\Pi_2$ . Profile plane  $P-\Pi_3$  with projection  $A_3B_3$  of segment AB is rotated about the axis of applicate OZ counter-clockwise from a positive direction +Z till its full coincidence with frontal plane  $F-\Pi_2$ . The constructed two-dimensional geometrical model of segment AB is shown to the right in Fig. (7).

A two-dimensional geometrical model is called as the complex drawing of a geometrical image [4, 6, 19-22].

For the European system of measurement in the two-dimensional complex drawing of a geometrical image, the horizontal projection is located below the frontal projection, and the profile projection is located to the right of the frontal projection (Fig. 7).

Constructed frontal projection  $A_2B_2$ , horizontal projection  $A_1B_1$  and profile projection  $A_3B_3$  of segment AB also satisfy the laws of projective connections 1-1, 2-2, 3-3 [4, 37].

Elements of the European system of measurement also correlated with different projections of a geometrical image as follows:

1. Coinciding ( $+X \equiv -Y$ ) the positive portion of the axis of abscissas OX and the negative portion of the axis of ordinates OY, and also coinciding ( $+Z \equiv -Y$ ) the positive portion of the axis OZ and the negative portion of the axis of ordinates OY limit frontal plane  $F-\Pi_2$  and the frontal projection of a geometrical image.

2. Coinciding ( $+X \equiv -Y$ ) the positive portion of the axis of abscissas OX and the negative portion of the axis of ordinates OY, and also coinciding ( $+Y \equiv -Z$ ) the positive portion of the axis of ordinates OY and the negative portion of the axis OZ limit horizontal plane  $H-\Pi_1$  and the horizontal projection of a geometrical image.

3. Coinciding ( $+Z \equiv -Y$ ) the positive portion of the applicate axis OZ and the negative portion of the axis of ordinates OY, and also coinciding ( $+Y \equiv -X$ ) the positive portion of the axis of ordinates OY and the negative portion of the axis of abscissas OX limit profile plane  $P-\Pi_3$  and the profile projection of a geometrical image.

For the realized way of construction of the complex drawing (Fig. 7) in the European system of measurement, the horizontal projection is the top view, and the profile projection is the left-side view.

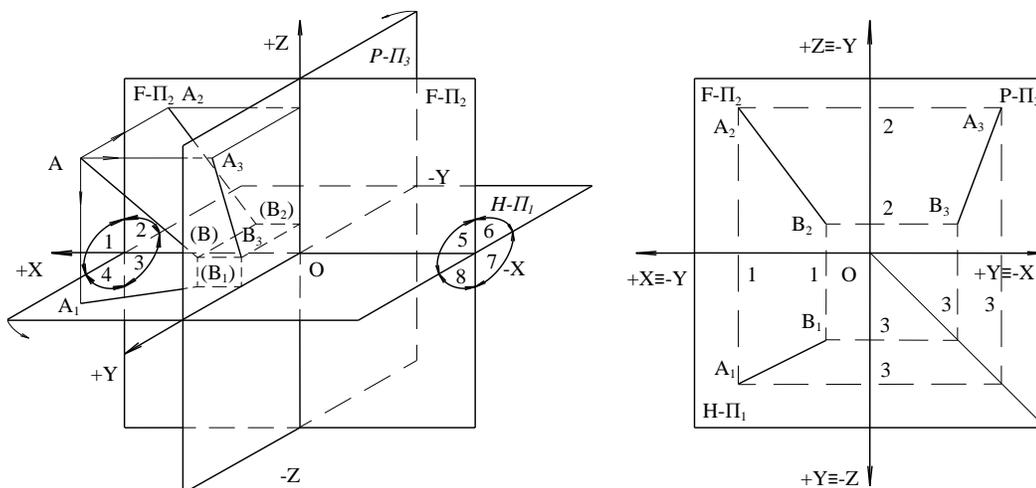


Figure 7: Geometrical models of segment AB of a straight line in the European system of measurement.

Hence, if planes of projections to consider opaque on the projective drawing of a product, the *top view* is located *below* the front view and the *left-side view* is located *to the right of the* front view. Such an arrangement of views corresponds to the *accepted* arrangement of views in the projective drawing of a product in the countries where the *European* system of measurement is used.

*Interrelations* of elements *American* (Fig. 6) and *European* (Fig. 7) systems of measurement with different projections of a geometrical image in the two-dimensional complex drawing *are not the same*. The differences result in the *various arrangements of views* in the projective drawing of a product.

An attempt to reach conformity of projections of an image (Fig. 6) and proper visualization of product views under the American standard (ISO A of ISO 5456) by simple turn of planes of projections in such *European* direction leads to loss of clearness of the drawing of a geometrical image (Fig. 8).

**4. CONCLUSIONS**

1. *Three mutually perpendicular planes*  $H-\Pi_1$ ,  $F-\Pi_2$ ,  $P-\Pi_3$  are considered as the *components of system of measurement* for geometrical modeling. The *right three-dimensional system of coordinates* OXYZ is allocated to determine the coordinates in these planes.

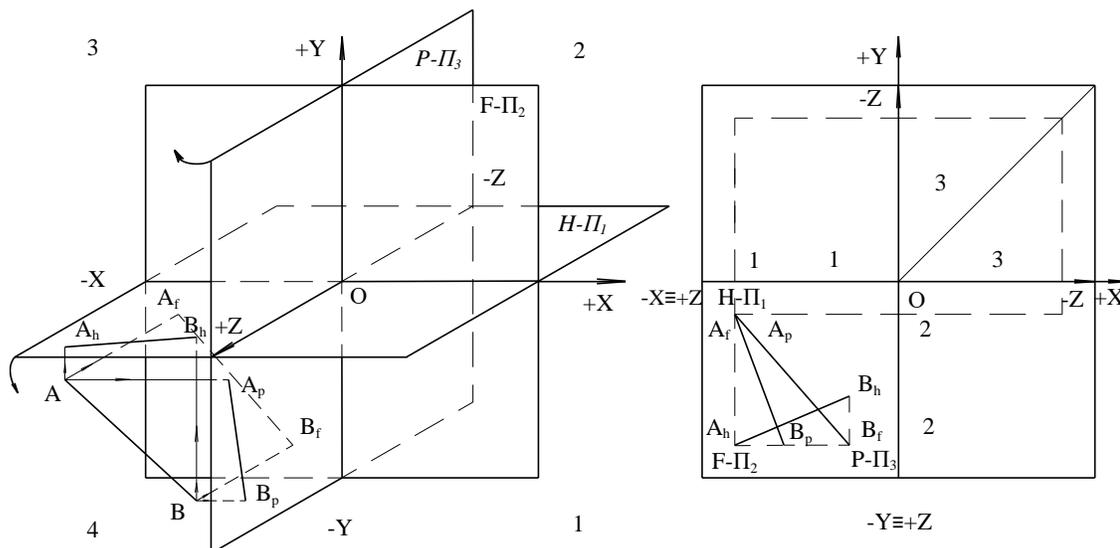
2. The *way of orientation* of the right three-dimensional system of coordinates OXYZ with respect to the set planes of projections  $H-\Pi_1$ ,  $F-\Pi_2$ ,  $P-\Pi_3$  is the

*first feature* of systems of measurement. In the *American* system of measurement, the axes OZ and OX coincide with *horizontal* plane  $H-\Pi_1$ , the axes OX and OY settle down in *frontal* plane  $F-\Pi_2$ , and axes OY and OZ belong to profile plane  $P-\Pi_3$  (Fig. 2). In the *European* system of measurement, the axis OZ and OX coincide with *frontal* plane  $F-\Pi_2$ , axes OX and OY settle down in *horizontal* plane  $H-\Pi_1$ , and axes OY and OZ also belong to profile plane  $P-\Pi_3$  (Fig. 4).

3. The *second feature* of systems of measurement for geometrical modeling is the *way of conditional division* of space into parts: semi spaces, quadrants, and octants. In the *American* system of measurement, the space is divided into four quadrants by horizontal plane  $H-\Pi_1$  and *profile* plane  $P-\Pi_3$  as shown in Fig. (3). In the *European* system of measurement, the space is divided into four quadrants by horizontal plane  $H-\Pi_1$  and *frontal* plane  $F-\Pi_2$  (Fig. 5).

4. The *third feature* of systems of measurement for geometrical modeling is the *order of numbering* of the allocated parts of space (semi spaces, quadrants, and octants). In the *American* system of measurement, this order shown in Fig. (3) differs from that used in the *European* system of measurement (Fig. 5).

5. *Interrelations* of elements *American* (Fig. 6) and *European* (Fig. 7) systems of measurement with different projections of a geometrical image in the two-dimensional complex drawing *are not the same*. The differences result in the *various arrangements of views* in the projective drawing of a product.



**Figure 8:** The rotation of the plane H into the vertical position performed in the other direction (down). The same holds for the profile plane P (Left).

An attempt to reach conformity of *projections* of an image (Fig. 6) and proper visualization of product views under the American standard (ISO A of ISO 5456) by simple turn of planes of projections in such *European* direction leads to loss of clearness of the drawing of a geometrical image (Fig. 8).

A way of elimination of *discrepancy of projections* of an image and views of a product for the *American* system we will consider in a separate research.

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